NOTES:

UNIVERSIDAD NACIONAL DE INGENIERIA
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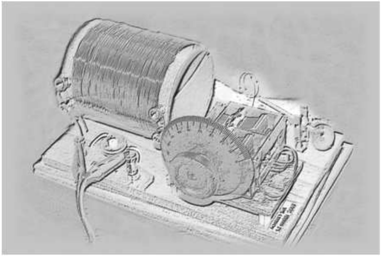
Contributions to Crystal Radio

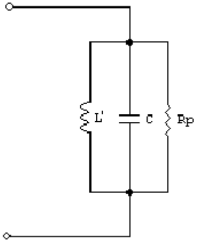
A web book by Ramon Vargas Patron



The above expression suggests that parallel resonance is between capacitor C and an inductor

$$L' = L\left(1 + \frac{1}{Q_s^{\,2}}\right) \quad \ldots(9)$$

Thus, the equivalent parallel resonant circuit is as shown in the figure below, for
frequencies in the vicinity of ω_0 .



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$$Z = R_p = \frac{R_s^2 + \omega_0^2 L^2}{R_s}$$

$$= \frac{R_s^2 (1 + Q_s^2)}{R_s}$$

$$= R_s (1 + Q_s^2) \quad \ldots(6)$$

where:

$$Q_s = \frac{\omega_0 L}{R_s} \quad \ldots(7)$$

is the inductor's Q factor at frequency ω0 .

Eq.(3) can be written as:

$$C(\omega_0^2 L^2 + R_s^2) - L = 0$$

or

$$CR_s^2 (Q_s^2 + 1) - L = 0$$

Then,

$$\frac{R_s^2 C}{L} = \frac{1}{Q_s^2 + 1}$$

Substituting into Eq.(4) yields:

$$\omega_0^2 = \frac{1}{LC}\left(\frac{Q_s^2}{Q_s^2 + 1}\right) \quad \ldots(8)$$

$$\omega_0^2 = \frac{1}{\left(1 + \frac{1}{Q_s^2}\right) LC}$$

Editor's Note:

The following chapters are important theoretical contributions to the understanding of Crystal Radio. Dr. Vargas' work delves deeply into the math behind the workings of these sets. The articles presented here reflect merely a small part of his larger work at the National Engineering University in Lima, Peru. I urgently recommend any interested reader to consult the original references and excellent web page of Dr. Vargas.

Original web location for book:
http://www.inictel-uni.edu.pe/index.php?option=com_content&view=article&id=235&Itemid=152

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$$= \frac{1 + j\omega C(R_s + j\omega L)}{R_s + j\omega L}$$

$$= \frac{1 - \omega^2 LC + j\omega R_s C}{R_s + j\omega L}$$

$$= \frac{(1 - \omega^2 LC + j\omega R_s C)(R_s - j\omega L)}{R_s^2 + \omega^2 L^2}$$

$$= \frac{R_s + j\omega(\omega^2 L^2 C + R_s^2 C - L)}{R_s^2 + \omega^2 L^2} \quad \text{...(2)}$$

Resonance is attained when the admittance function´s phase is zero. This is, when:

$$\omega^2 L^2 C + R_s^2 C - L = 0 \quad \text{...(3)}$$

The resonant frequency is given then by:

$$\omega_0^2 = \frac{1}{LC}\left(1 - \frac{R_s^2 C}{L}\right) < \frac{1}{LC} \quad \text{...(4)}$$

The admittance at resonance is:

$$Y = \frac{R_s}{R_s^2 + \omega_0^2 L^2} \quad \text{...(5)}$$

The input impedance at resonance is Z = 1 / Y. Then:

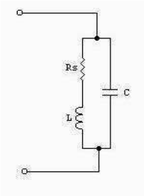
occurring, L' tends to the value L, so the tuned frequency changes. A new regeneration level implies a new value for L'.

Selecting a low value for Rg' = Rg // Rs will help reducing frequency detuning due to Ro variations. Bias issues usually impose constraints on the possible values for the source resistor Rs. So we are forced to minimize Rg, either by reducing Rr or selecting lower values for Rp. It´s much easier (and less expensive) to change the attenuator's total resistance than changing a complete antenna-ground system looking for a lower Rr.

APPENDIX

Type-I Lossy L-C Resonant Circuit

Consider an L-C parallel resonant circuit with series losses in the inductive branch.



The input admittance Y is given by:

$$Y = \frac{1}{R_s + j\omega L} + j\omega C \quad \ldots(1)$$

Crystal Receivers for the MW AM Band
By Ramon Vargas Patron
http://www.inictel-uni.edu.pe/index.php?option=com_content&view=article&id=235&Itemid=152

In this section we´ll focus on simple crystal radios for the 540kHz to 1650kHz AM broadcasting band. We shall start with the very basic ones and then progress towards the amplified versions. The designs will make use of a coil wound on a cylindrical or rectangular ferrite bar, the type found in portable AM receivers. This coil receives the generic name of ferrite antenna. It has the effectiveness of an air-cored antenna coil coupled to an outside aerial of several meters long.

Our first schematic diagram (Fig.1) refers to a basic crystal set using a pair of 2000-ohms DC-resistance magnetic headphones, with a series connected parallel R-C network. This network reduces the loading imposed by the headphones on the tank circuit, increasing the selectivity of the receiver, or the ability to “separate” adjacent stations. This will be highly appreciated by an operator trying to tune-in a weak signal with a strong transmitter occupying an adjacent channel.

The variable capacitor we shall use is a salvaged 500pF unit, usually a nominal 475pF double-gang capacitor (only one gang will be required). The ferrite antenna consists of 50 close-wound turns of Litz wire of a gauge similar to that found in home portables (we can also use solid enamelled copper wire #26 or #28 AWG). Prior to winding the coil, it is advisable to wrap the ferrite bar with thin cardboard or two layers of paper, in order to protect the wire from damage due to friction with the bar. Turns may be held in place using wax or some kind of adhesive tape. Around 185uH of inductance will be needed for

$$R_0 = (\mu+1)(R_g \,//\, R_e) + \frac{1}{h_{oe}} \qquad , \mu = g_m / h_{oe}$$

The formula for Rg remains the same.

Explanation for why the receiver detunes when the gain control is adjusted The tuning tank has series losses, mostly attributed to the coil, if the tuning capacitor is of a high-quality type. The lossy coil L can be converted into an equivalent ideal lossless coil L’ with a parallel loss rp:

$$L' = L\left(1 + \frac{1}{Q_s^{\,2}}\right)$$

$$r_p = r_s\left(1 + Q_s^{\,2}\right)$$

where

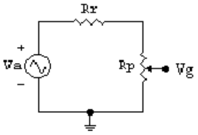
$$Q_s = \frac{\omega_0 L}{r_s} = \frac{2\pi f_0 L}{r_s}$$

The coil L’ now tunes with the capacitor to the same frequency ω0 the lossy coil was tuning in conjunction with the cap:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{L'C}}$$

When a coil has series losses, the tuned frequency is affected and will be given by a different formula. As can be seen L’ is greater than L, so, given C, the lossy inductor L will tune to a lower frequency than that obtained with a lossless L.

Now, the total parallel loss is rpt = Ro // rp. As has been seen, a manual change in the RF stage’s voltage gain will cause variations in the output resistance Ro of the amplifier. Hence, the net parallel loss rpt will change, driving us to accommodate regeneration values to the new situation. Regeneration partially cancels out parallel loss, which is a representation of the original series losses of the coil L. When cancellation is



Antenna, losses and attenuator

Fig.4 Antenna and input equivalent

Numerical examples

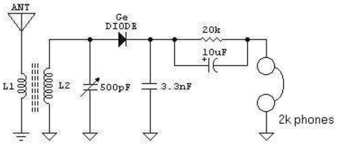
The following examples assume a JFET having the following small-signal mid-band parameters: gm = 4mS, RD = 40k ohms, μ = gm RD = 160. The antenna-ground system loss resistance plus the antenna's radiation resistance is Rr = 100 ohms. The source resistor is Rs = 1k ohms. The input attenuator's total resistance is Rp = 0.47k ohms and the output load's impedance is resistive and equal to RL = 40k ohms.

K	$\frac{KR_p}{R_r + R_p}$	Rg	Av	Av_T	Ro
0.1	0.082	0.043kΩ	71.27	5.84	46.60kΩ
0.25	0.206	0.0933kΩ	62.84	12.94	53.68kΩ
0.5	0.412	0.138kΩ	56.86	23.43	59.48kΩ
0.75	0.618	0.134kΩ	57.35	35.44	59.02kΩ
1	0.824	0.082kΩ	64.55	53.19	52.24kΩ

Modifications of formulae for the bipolar case

The bipolar case (in a common-base configuration) requires that μ be made equal to gm / hoe and 1/hoe substituted for RD. Also, Re should be substituted for Rs . So we now have:

this coil to tune down to 540kHz with the said variable capacitor. We can trim the inductance sliding the cardboard or paper cylinder along the ferrite bar. The detector diode is a germanium device and types 1N34, 1N60, AA119, etc. may be used.



L1: 4 turns of Litz wire (see text).
L2: Main tuning coil - 50 turns of Litz wire (see text).
⏚: Circuit´s common ground
The diode detector is a germanium type.

Fig.1 Basic crystal receiver

Strong locals within 5km from our receiver's site should be heard through the headphones, especially if these are of a sensitive type. Medium strength locals require that our hearing room be a quiet place. Sometimes local weather will give us surprises. We may also find that rotating horizontally the ferrite bar increases the volume of our signals or decreases them, due to the ferrite antenna´s directivity. This may help reject interference from the strong ones when hearing a weak signal. If our site is at respectable distance from local transmitters (well, for our basic receiver that means more than 5km), an inverted-L outdoor antenna may be tried for improved reception. A horizontal span of at least 10 meters and a height of 5 meters minimum above obstructions will help. An additional coil will be needed for connection of the antenna. Four turns of Litz or solid enamelled copper wire, whatever has been used for the main tuning coil, will do. This winding should be separated 2.5cm (1”) from the “cold” end of the main coil (the end connected to the circuit’s common ground). One end of the small coil connects to the antenna; the other end should connect to a good ground system (connection to a cold-water pipe using a suitable clamp could be tried).

If a pair of 2000-ohms magnetic headphones is not available, a piezoelectric crystal or ceramic earphone paralleled by a 68k ohms resistor can be used instead. For this alternative, we discard the 3.3nF (0.0033uF) capacitor and the parallel 20k / 10uF network. Connections should be made as indicated by Fig.2 below. This earphone is a high-impedance type, very sensitive and should not be confused with the small low-impedance dynamic type commonly found in “walk-man” players.

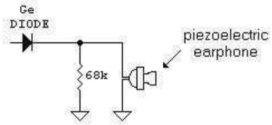


Fig.2 Connection of a piezoelectric crystal or ceramic earphone

Usually, loose-coupling an efficient antenna to the tuning tank renders a sensitive receiver having acceptable selectivity. The R-C network used in the basic crystal receiver discussed above also helps to improve selectivity. This idea has been in use for some years now and is a sample of what can be done at the

Vo in terms of Vg yields an expression for the voltage gain Av = Vo / Vg:

$$A_v = \frac{V_0}{V_g} = \frac{(\mu+1)R_s R_L}{(R_s + R_g)\left[(\mu+1)(R_g // R_s) + R_D + R_L\right]}$$

which can also be written as:

$$A_v = \frac{(\mu+1)R_L}{\left(1+\frac{R_g}{R_s}\right)\left[(\mu+1)\frac{R_g}{\left(1+\frac{R_g}{R_s}\right)} + R_D + R_L\right]}$$

The input resistance of the stage as seen towards the JFET’s source may be found to be:

$$R_{in} = \frac{V_g'}{(-I_D)} - R_g' = \frac{R_D + R_L}{(\mu+1)}$$

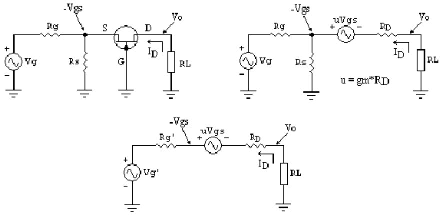
For the case the generator is an antenna, if we let Va be the voltage induced on the antenna by the radio waves, Rr the antenna-ground system’s resistive losses plus the antenna’s radiation resistance, Rp the variable input attenuator’s total resistance and K the fraction of this resistance existing between the slider and the ground end, we get (Fig. 4):

$$V_g = V_a \frac{KR_p}{R_r + R_p}$$

$$R_g = \left[R_r + (1-K)R_p\right] // KR_p$$

The voltage gain from generator to load is :

$$Av_T = \frac{V_0}{V_a} = \frac{V_0}{V_g} \cdot \frac{V_g}{V_a} = \frac{KR_p}{R_r + R_p} A_v$$



Small signal modelling of RF JFET stage

Fig.3 Small-signal models

In the figure above,

$$R_g' = R_g // R_s = \frac{R_g R_s}{R_g + R_s}$$

$$V_g' = V_g \frac{R_s}{R_s + R_g}$$

Solving for ID gives:

$$-I_D = \frac{(\mu+1)V_g'}{(\mu+1)R_g' + R_D + R_L} = (\mu+1)V_g \frac{R_s}{R_s + R_g} \cdot \frac{1}{(\mu+1)R_g' + R_D + R_L}$$

Knowing that Vo = Io RL, where Io = -ID, we arrive at:

$$V_0 = (\mu+1)V_g \frac{R_s}{R_s + R_g} - I_0\left[(\mu+1)R_g' + R_D\right]$$

The output resistance is then:

$$R_0 = (\mu+1)R_g' + R_D = (\mu+1)(R_g // R_s) + R_D$$

headphones’ end to enhance the ability of our radio to separate signals. In recent years, attention has also been given to the use of audio transformers in crystal sets to step-up the rather low headphone impedances to a larger value that will better match the audio source resistance, resulting in more volume from the headphones, while still maintaining a good selectivity. The audio source resistance is actually the diode´s output audio resistance and is a result of the detection action taking place in the radio, whereupon radio frequency energy is converted into audio frequency signals.

Fig.3 shows a crystal receiver using an audio transformer to couple the headphones to the circuit. A transformer has two windings, the primary and the secondary. N is the turns-ratio, or ratio of the voltage impressed across the primary to the voltage obtained across the secondary. Transformers also change impedance levels, so sometimes they are specified by their impedance transformation ratio or N2:1.

The detector diode´s small-signal output audio resistance ranges from around 40k ohms to about 150k ohms for available germanium diodes. Diodes having the higher resistances are preferred for maximum sensitivity and selectivity. This means that weaker signals will be detected and less loading will be imposed on the tuning tank. Accordingly, greater impedance transformation ratios will be required to match the headphone’s impedance to the diode’s.

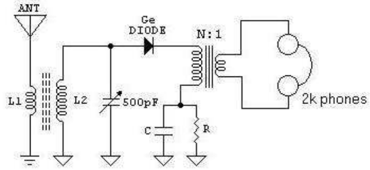
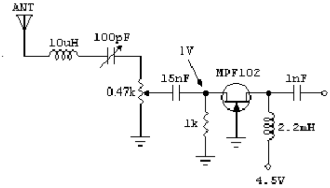


Fig.3 Crystal receiver using a coupling audio transformer

Magnetic headphones have a DC resistance of around 2k ohms and an average AC impedance that is around six times this value, or 12k ohms. Strictly speaking, AC impedance varies with frequency, so we refer to a measured average value over the 300Hz~3300Hz audio range. If our diode has, say, 60k ohms for its audio output resistance, then we will need a transformer having an impedance transformation ratio of 60k:12k, or 5:1. A word of caution here: the transformer should be a type that will work with the said impedance levels.

We have commented the usefulness of the R-C network when looking for selectivity improvements. The philosophy behind says that resistor R should be more or less equal to the headphones' average AC impedance, and capacitor C should be selected with a value that will by-pass audio signals. The capacitor's reactance at 300Hz should be no more than R/10. Values are not so critical.

When using an audio transformer for impedance matching (term meaning that we are trying to make the transformed load impedance equal the source resistance) an R-C network of the



RF stage using the MPF102 JFET

Fig.2 JFET-based RF stage

The small-signal models employed for gain and impedance calculations can be seen in Fig.3. Vg is the generator's signal voltage and Rg is the generator's output resistance. RD is the drain's dynamic resistance and Rs is the source's bias external resistor. The JFET's small-signal model uses a vacuum tube-like circuit model. Hence, we need defining μ as the amplification factor, being μ = gm RD. The quantity gm is the JFET's forward transfer conductance for frequencies in the mid band.

Fig.2 shows the schematic of the RF stage. The 10uH RF choke in series with the antenna lead-in effectively blocks FM / TV interference and a power grid-like annoyingly strong interference at the author's location. The series connected 100pF variable capacitor helps attenuating very strong signals and will also improve selectivity.

The output impedance of the RF stage depends on the impedances connected to its input. Having some gain control through the use of a variable input attenuator is advisable, given the dynamic range of available SW radio signals. However, this will bring about changes in the output impedance of the amplifier, which in turn will cause some receiver detuning.

In order to have some quantitative knowledge of the factors affecting the output impedance of the amplifier, the author conducted some calculations assuming operation in the mid band, so the device's parameters could be considered real quantities. For higher frequencies complex numbers would have to be used. However, the theoretical results obtained were coherent with the experimental observations at SW frequencies.

type described will be necessary. It should be connected as shown in Fig.3. Resistor R should be made equal to the transformed load impedance, as seen from the primary side of the transformer. With reference to the example above, R should be 60k ohms and an adequate value for C would be 0.1uF.

Before going into the group of circuits featuring post-detection audio frequency (AF) amplification, let´s take a look to the configuration shown in Fig.4. The germanium diode paralleling the 2000-ohms headphones appears to be floating above ground. Apparently there is something wrong with the schematic. However, if we realize that something must be coupling radio energy from the diode's cathode end to the circuit's common ground, permitting proper operation of the receiver, we will be on our way into solving the mystery.

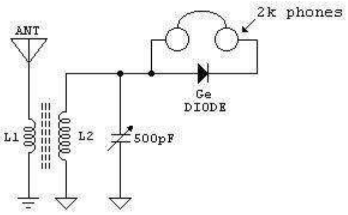
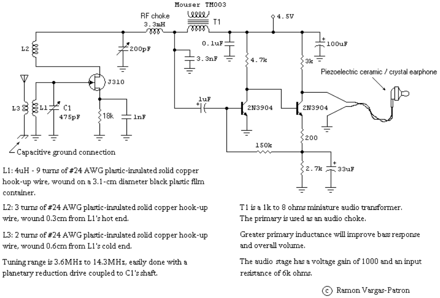


Fig.4 Floating-diode crystal receiver

And the answer is that…….naturally occurring stray capacitance from headphones and diode's cathode to common ground closes the circuit for radio frequency currents, permitting detection to take place in the usual way. So we happily get AF currents flowing through the headphones.

Now we will give a couple of examples of crystal sets featuring AF amplification. The first schematic (Fig.5) shows a receiver that uses a germanium AF transistor. Germanium amplifying devices exhibit at room temperature relatively large values of leakage collector current, or Iceo. PNP transistors find this current just sufficient for their amplification factor β, or HFE, to build up to a useful level. This is the case of European types AC125, AC126, AC188 and others. This is also certain for many US and Japanese types, such as 2N109, 2N188A, GA53677,……, 2SB22, 2SB370, 2SB496,……. The list is rather long. The good news is that we can dispense with the usual base bias resistive network, and still make a very simple amplifying stage. The addition of AF amplification will give our basic receiver extra volume. It should be noticed that there is no DC path between the diode and the transistor's base. The 100k ohms potentiometer is a volume control.

Our second amplified receiver (Fig.6) makes use of a 2N3904 silicon NPN transistor. We need external base bias for our amplifying device this time, as silicon transistors exhibit much less collector leakage currents. The 200k ohm resistor biases the transistor's base so collector to ground voltage is around 1 volt. We also need here to isolate the base from the potentiometer for DC currents. We use the 0.1uF capacitor for this purpose. We must invert the 1.5V battery's polarity for this circuit, as currents in NPN transistors flow in opposite direction to that of PNP devices.



Armstrong-type SW regenerative receiver

Fig.1 SW regenerative receiver

Fig.1 shows the schematic of the author's initial SW receiver prototype. Testing for better sensitivity, a very simple untuned RF amplifier using the MPF102 N-channel JFET in common-gate configuration was added to the J310-based SW receiver. This stage was capacitively coupled to the tuning tank of the regenerative detector. In order to take advantage of the maximum available voltage gain a 2.2-mH RF choke was connected between the JFET's drain electrode and the power supply. It was found that tight coupling to the detector stage would give the best noise / interference rejection. A low distributed-capacitance three-section pie-wound RF choke was selected for this application. Low-cost pile-wound chokes affected the upper end of the tuned band, precluding their use.

Small Signal Calculation of a SW RF Stage

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Our article "The Modern Armstrong Regenerative Receiver" presented a 530kHz~1700kHz MW regenerative detector based on the J310 N-channel JFET as a counterpart of the famous vacuum-tube homologous. The receiver there described was found to be very selective and sensitive. Tests conducted with different coil arrangements and an outside aerial suggested also excellent performance throughout the entire short wave spectrum.

When testing the receiver, the planetary reduction drive used in conjunction with the 475-pF broadcast variable proved to be very useful when tuning adjacent stations in the crowded SW bands. Although the author's SW prototype worked impressively well, some variations were devised thinking of experimenters wishing to replicate the receiver but lacking maybe space for a decent antenna. Duly attention was given for a minimum parts count.

There is the saying that a working receiver will be as good as its associated antennaground system. Situations exist where local conditions will render an outside randomwire antenna useless for sending good signals to a receiver. In this case, an RF-amplifier stage ahead of the regenerative detector will give the extra voltage gain required for a successful performance.

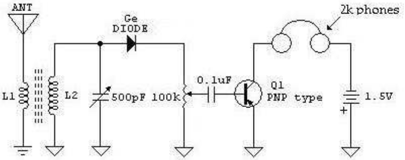

Fig.5 Crystal receiver with AF amplification using a germanium transistor

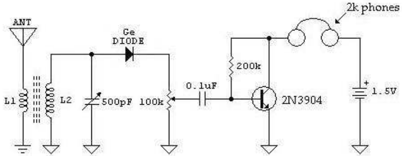

Fig.6 Crystal receiver with AF amplification using a silicon NPN transistor

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Notes on the Demodulation of AM Signals

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A DSBC (double side band plus carrier) signal may be expressed (Fig.1) as:

f(t) = A [1 + m.g(t)] cosωct …(1)

where:

g(t) = modulating signal
m = modulation index ($0 < m \leq 1$)
ωc = 2πfc
fc = carrier frequency
A = amplitude of unmodulated carrier

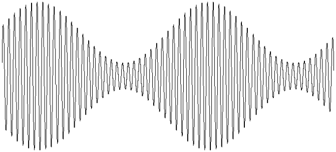

Fig.1 DSBC AM signal

Let:

g(t) = cosωmt
ωm = 2πfm
fm = frequency of the modulating signal

Then:

2. http://hjem.get2net.dk/helthansen/regenerative_vacuum_valve.htm

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January 19th, 2006

relaxation oscillator for high regeneration levels. This is an unwanted feature.

This said, “grid-leak”- type of detection is virtually cancelled. A VERY strong signal would be needed to overcome the gate-source bias and make the corresponding junction to conduct. Detection now is of the square-law type. The non-linear characteristics of the JFET make this possible. The “grid” R-C components have no effect on the detection action and have been maintained there for nostalgic reasons only. The receiver seems to be a bit more selective if these components are left in the circuit, although no explanation has been found for this.

The modulation is extracted from the junction of the RF choke and the primary winding of the Mouser TM003 audio transformer acting as an audio choke. The 3.3-nF capacitor, also a Mylar type, filters any residual RF component leaking through the 3.3- mH RF choke and that may be entering into the AF amplifier.

The two-transistor amplifier has a voltage gain of approximately 1000 and has an input resistance of 6k ohms at 1kHz. A piezoelectric ceramic / crystal earphone gives comfortable listening.

One final comment is that a ground plane underneath the circuit’s layout is needed for stable and hand capacitance-free operation. Connect the circuit’s common ground to this plane.

References

1. Edwin H. Armstrong’s “Wireless Receiving System”, US Patent 1,113,149

$$f(t) = A\,[1 + m.\cos\omega_m t\,]\cos\omega_c t \quad ...(2)$$

which may be written as:

$$f(t) = A\cos\omega_c t + \frac{mA}{2}\cos(\omega_c + \omega_m)t + \frac{mA}{2}\cos(\omega_c - \omega_m)t \quad ...(3)$$

1. Square-law detection of DSBC signals

The output of a square-law detector is of the form:

y = x2 …(4)

where x = input signal.

Vaccum-tube diodes and semiconductor (solid state) diodes have this type of response for small inputs.

Substituting eq. (3) in (4) yields:

$$f^2(t) = A^2\cos^2\omega_c t + \frac{m^2A^2}{4}\cos^2(\omega_c + \omega_m)t + \frac{m^2A^2}{4}\cos^2(\omega_c - \omega_m)t$$
$$+ mA^2\cos\omega_c t\cdot\cos(\omega_c + \omega_m)t + mA^2\cos\omega_c t\cdot\cos(\omega_c - \omega_m)t$$
$$+ \frac{m^2A^2}{2}\cos(\omega_c + \omega_m)t\cdot\cos(\omega_c - \omega_m)t$$

which may be arranged as:

$$f^2(t) = \frac{A^2}{2}(1 + \cos 2\omega_c t) + \frac{m^2A^2}{8}[1 + \cos 2(\omega_c + \omega_m)t] + \frac{m^2A^2}{8}[1 + \cos 2(\omega_c - \omega_m)t]$$
$$+ \frac{mA^2}{2}[\cos(2\omega_c + \omega_m)t + \cos\omega_m t] + \frac{mA^2}{2}[\cos(2\omega_c - \omega_m)t + \cos\omega_m t]$$
$$+ \frac{m^2A^2}{4}[\cos 2\omega_c t + \cos 2\omega_m t]$$

…(5)

The output of the square-law detector contains AF and RF components. After filtering out the latter we are left with:

$$D(t) = mA^2 \cos\omega_m t + \frac{m^2 A^2}{4}\cos 2\omega_m t \quad \ldots(6)$$

The first term resembles the modulation. The second term constitutes a distortion component.

2. Product (synchronous) detection

If we multiply eq. (3) by the term s(t) = B cosωct, we obtain an expression for the output of a product or synchronous detector, whose schematic representation is given in Fig.2.

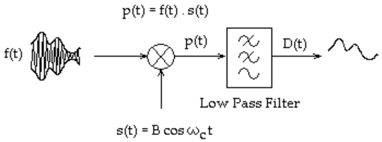

Fig.2 Product (synchronous) detector

The output of this type of detector is given by p(t) = f(t) . s(t), this is,

$$f(t)\cdot s(t) = AB\cos^2\omega_c t + \frac{mAB}{2}\cos\omega_c t\cdot\cos(\omega_c+\omega_m)t + \frac{mAB}{2}\cos\omega_c t\cdot\cos(\omega_c-\omega_m)t$$

which may be written as:

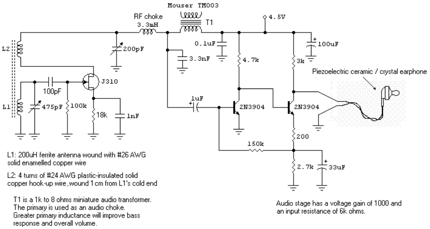

Fig.3 Armstrong-type MW regenerative receiver using the J310 JFET

A vacuum tube is a robust electron device. It may stand reasonable signal overloading and some abuse also. Solid-state devices are delicate and need protection circuitry. The J310 JFET of Fig.3 is not an exception. We need to include a source resistor for protection and biasing reasons. Without this resistor the quiescent current would be too large, some 30mA according to the manufacturer. Really too big for our purposes. The gate-source junction would be also vulnerable to very strong signals that eventually could reach our receiver.

In the figure above, the 18-kohm source resistor regulates the drain current ID to 0.157mA when the power supply is 4.5 Volts DC. So we get 2.83 Volts for the voltage drop across the resistor. The gate-source bias-voltage is then -2.83 Volts. For a 9-Volt supply, the JFET's drain current increases only to 0.16mA. We get a very steady operating-point current with respect to power supply variations. The resistor needs RF decoupling and that is the reason for the 1nF capacitor. It is recommended a Mylar type. Greater capacitance values are not recommended, as they can make the circuit operate as a should be much greater than the carrier's period T. For example, for the MW AM broadcast band, the lowest tuned frequency is 530kHz. For this case, T = 1.887E-06 sec, or 1.887 µsec. Clearly, t >> T. However, t can not be made to be too large, or failure to follow distortion will occur in the recovered audio. If TA is the period of the highest modulation frequency, then the inequality TA > t should be accomplished. Let 3kHz be the maximum modulation frequency component. Then, TA = 3.33E-04 sec = 333 µsec, which is greater than t = 100 µsec. Thus, the selected values for Rg and Cg are correct. Due to an important property of the voltage-clamp circuit, the tuning tank sees a parallel load approximately equal to Rg/3, or 333.33k ohms in our case.

The solid-state Armstrong regenerative receiver

The Armstrong circuit has also its solid-state counterpart, and historically speaking, it has been devised using bipolar transistors as well as junction FETs (JFETs) and insulated-gate FETs (IGFETs or MOSFETs).

There is an important difference in the detection action of the solid state version. We shall base our comments on the Armstrong receiver shown in Fig.3 that will tune the 530kHz~1700kHz MW AM broadcast band.

$$f(t)\cdot s(t)=\frac{AB}{2}(1+\cos 2\omega_c t)+\frac{mAB}{4}\left[\cos(2\omega_c+\omega_m)t+\cos\omega_m t\right]$$
$$+\frac{mAB}{4}\left[\cos(2\omega_c-\omega_m)t+\cos\omega_m t\right] \qquad (7)$$

After filtering out RF components we obtain:

$$D(t)=\frac{mAB}{2}\cos\omega_m t \qquad \ldots(8)$$

and the distortion component of frequency 2ωm does not exist.
3. Single side band (SSB) detection
An SSB signal may be expressed by:

$$f(t) = A\cos(\omega_c + \omega_m)t$$

or …(9)

$$f(t) = A\cos(\omega_c - \omega_m)t$$

the plus sign used for an upper side band (USB) signal and the minus sign for a lower side band (LSB) signal. Product and mixer-type demodulators are used for SSB detection.

3.1 Product detection of SSB signals

Consider a product detector with inputs:

$$f(t) = A\cos(\omega_c + \omega_m)t$$

and

$$s(t) = B\cos\omega_c t$$

The detector performs the mathematical function:

$$p(t)=f(t)\cdot s(t)=AB\cos\omega_c t.\cos(\omega_c+\omega_m)t$$

which is identical to:

$$p(t) = \frac{AB}{2}\left[\cos(2\omega_c + \omega_m)t + \cos\omega_m t\right]$$ …(10)

Clearly, if we filter out RF components from p(t) we get:

$$D(t) = \frac{AB}{2}\cos\omega_m t$$ …(11)

which is the desired modulation signal.

3.2 SSB detection using mixer-type demodulators

Here, a SSB signal at IF frequencies is mixed with the output of a beat frequency oscillator (BFO). This permits modulation retrieval.

Let f(t) = A cos(ωc + ωm)t be the SSB signal and s(t) = B cos(ωc + Δω)t the output from the BFO. The mixer's output is:

$$p(t) = [f(t) + s(t)]^2$$ …(12)

Then:

$$p(t) = \left[A\cos(\omega_c + \omega_m)t + B\cos(\omega_c + \Delta\omega)t\right]^2$$

or equivalently,

$$p(t) = \frac{A^2}{2}\left[1 + \cos 2(\omega_c + \omega_m)t\right] + \frac{B^2}{2}\left[1 + \cos 2(\omega_c + \Delta\omega)t\right] + AB\left[\cos(2\omega_c + \omega_m + \Delta\omega)t + \cos(\omega_m - \Delta\omega)t\right]$$ …(13)

After removal of the RF frequency components from the output of the mixer we are left with:

$$D(t) = AB\cos(\omega_m - \Delta\omega)t$$ …(14)

The final result is that a negative average voltage equal to the carrier's amplitude develops across the triode's input. If the carrier is amplitude modulated, the average voltage follows the modulating signal. What is interesting here to note is that in the absence of a carrier the control grid-to-cathode voltage is almost zero. After carrier detection, the average voltage turns negative, so there will be a drop in the average plate current. If the carrier is amplitude modulated, the average plate current will decrease in more or less degree, following the modulation.

It should be clear that an amplified version of the modulating signal exists at the tube's output in the form of low-frequency variations of the plate current. An RF choke prevents the high-frequency components of this current from circulating through the high-impedance headphones, which are acting as an audio load (Fig.1).

However, RF-choking action is not always 100% efficient, and some RF energy may leak into the headphones or following AF stages. This could give rise to some very strange effects, such as low-frequency motorboating at some value of the throttle capacitor, or the receiver breaking into oscillation when the throttle capacitor's vanes are fully unmeshed. Usually, selecting an RF choke with enough inductance will keep these problems away. A decoupling capacitor may also help. In Fig.1, capacitor C3 bypasses any residual RF components, leaving only AF currents flowing through the headphones. For grid-leak detection to work properly, the grid-circuit time constant:

t = Rg x Cg = 1M x 100pF = 0.0001 sec = 1E-04 sec = 100 μsec

Usually, this occurs when the circuit is in the threshold of oscillation.

Regenerative receivers need rather small operating-point currents, and it is not unusual for them to operate satisfactorily with low plate voltages. Low currents also make regeneration control smoother.

AM demodulation is accomplished through grid-leak detection. Fig.2 shows the key to understanding this type of detection. First, the triode's input is modelled as a diode pointing downwards. When the grid turns positive with respect to the tube's cathode, due to the presence of the positive half cycle of the carrier, some electrons emitted by the cathode are attracted by the grid, flowing in the external circuit to the grid capacitor. As a result, the grid capacitor replenishes its charge. During the carrier's negative half cycle, conduction between cathode and grid stops and charge leaks from the capacitor through the external circuit (L1 and the grid resistor). The next incoming carrier cycle the phenomenon repeats itself. The combination of Cg = 100pF, Rg = 1M and the equivalent diode act as a voltage-clamp circuit.

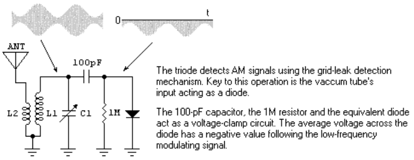

Fig.2 L-C tuning tank and equivalent input voltage-clamp circuit

We observe that the spectrum of the modulating signal has been recovered shifted down in frequency by an amount $\Delta\omega/2\pi$ Hertz. There the importance of the correct adjustment of the BFO.

4. Detection of DSBC signals using a regenerative receiver in the oscillating mode

Usually DSBC signals are detected in regenerative receivers adjusting the regenerative gain slightly below the oscillation point, the nonlinearities of the active device being responsible for the demodulation process, usually of a square-law type.

However, in the oscillating mode, DSBC signal detection is also possible. When tuned to the same frequency, a gently oscillating regenerative receiver will lock onto an incoming carrier. Both signals will be present across the tank circuit and hence will be mixed by the active device.

If A is the amplitude of the unmodulated carrier and B is that of the oscillation across the tuned circuit (having the same frequency as the carrier), then, using eq.(2) we get:

$$f(t) = A\,(1+ m.\cos\omega_m t)\,\cos\omega_c t + B\,\cos\omega_c t \quad ...(15)$$

for the signals across the tank circuit.

Eq.(15) may be written as:

$$f(t) = (A+B)\cos\omega_c t + \frac{mA}{2}\cos(\omega_c + \omega_m)t + \frac{mA}{2}\cos(\omega_c - \omega_m)t$$

...(16)

After square-law detection and filtering we are left with the recovered modulation and an unwanted distortion term:

$$D(t) = mA(A+B)\cos\omega_m t + \frac{m^2A^2}{4}\cos 2\omega_m t$$ …(17)

Usually, harmonic distortion levels are tolerable.

5. Demodulation of SSB signals in a regenerative receiver

Detection of SSB signals requires the receiver to be working in the oscillating mode. Strong oscillations are needed so the receiver doesn´t lock to the incoming signal. Otherwise, the recovered audio will have the known "quack-quack" type of sound. Alternatively, input signals can be attenuated by the operator.

Detection is, again, of the square-law type, after a mixing process carried out by the nonlinearities of the active device.

If A $\cos(\omega c + \omega m)t$ is the SSB signal and B $\cos(\omega c + \Delta\omega)t$ the receiver's oscillation, both across the tank circuit, the mixer's output will be:

$$p(t) = \left[A\cos(\omega_c + \omega_m)t + B\cos(\omega_c + \Delta\omega)t\right]^2$$ …(18)

yielding after filtering out RF frequency components:

$$D(t) = AB\cos(\omega_m - \Delta\omega)t$$ …(19)

Again, careful tuning of the receiver will be necessary so that $\Delta\omega \rightarrow 0$. Also, the oscillation's frequency should be very stable.

Lima-Peru, South America
August, 21st, 2005

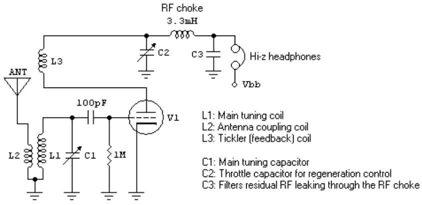

Fig.1 Basic Armstrong Regenerative Receiver

With reference to this circuit, the passing radio waves induce a voltage on the antenna. Induced currents flow through L2, which magnetically couples the RF energy to the L1- C1 tuned circuit. The grid circuit elements, Cg =100pF and Rg = 1M couple the tuned radio signal to the triode's input. Amplified RF currents flow in the plate circuit, setting up a magnetic field around L3 which couples energy back into the tank circuit in phase with that imposed by the radio wave, reinforcing it. Now we get stronger RF currents across L3, more energy fed back in phase, again an amplification……

If enough energy is fed back and enough amplification is obtained from the triode, the circuit will break into oscillation. For DSBC (double side band with carrier) AM demodulation, we don´t want the receiver oscillating. The mission of the throttle capacitor C2 is to limit the amount of current flowing through L3 so that the circuit won´t oscillate. C2 is adjusted so we get maximum amplification of the incoming radio signal.

The Modern Armstrong Regenerative Receiver

By Ramon Vargas Patron
http://www.inictel-uni.edu.pe/index.php?option=com_content&view=article&id=235&Itemid=152
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Lee de Forest's invention of the Audion in 1906 led to marvelous developments in radio receiver technology. But it was not until Major Edwin H. Armstrong conducted a thorough investigation of the principles of operation of the new three-electrode tube that the technological jump took place. Mr. Armstrong applied in 1913 for a patent on the regenerative receiver, one of the most famous radio inventions, maintaining a long litigation in the courts with the inventor of the Audion. However, he managed to develop a large number of radio circuits utilizing the principle of regenerative amplification or equivalently, positive feedback in amplification circuits.

The pioneering work of Major Armstrong on the regenerative receiver has come to our days in the form of the so called Armstrong circuit, the most popular receiver used by experimenters and Hams throughout the world. Its most basic representation is depicted in Fig.1 below.

On the Reduction of Detector Diode Losses in a Crystal Radio

By Ramon Vargas Patron
http://www.inictel-uni.edu.pe/index.php?option=com_content&view=article&id=235&Itemid=152
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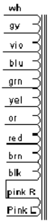
Current design techniques for high-performance crystal radio receivers call for the use of quality components in the RF, detection and AF sections of these radios, together with overall good antenna-ground systems. Sensitive magnetic or piezoelectric headphones, low-loss audio matching transformers and R-C equalization networks, or "bennies", are mandatory in AF ends. RF stages require high-Q coils and low-loss fixed or variable capacitors. Interesting to note here is the fact that varactor diodes are finding widespread use as replacements for capacitors of the variable type in a number of shortwave crystal radio designs. Detector diodes, on the other hand, should be high-quality sensitive types, usually germanium devices optimized for radio frequency operation, i.e., having small reverse currents and improved detection efficiency. Very good performance has been reported [1] employing Schottky-barrier diodes in the detection stages, although these require the use of rather expensive large turns-ratio audiomatching transformers for optimum power transfer to the operator's headphones.

A simple feedback technique using only passive circuit components can improve the efficiency of a cheap low-quality germanium detector diode. For this means, a fraction of the detected RF energy is directed back to the tuned L-C circuit, in



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3. Bringhurst, Steve "S-T-M Calrad / Bogen 'Select to Match' Impedance Matching
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September 9th 2004

phase with the incoming signal, increasing thereby the loaded tank's Q. This subtlety has been recently studied and tested by the author in a ferrite-antenna based simple crystal set, using a number of germanium diodes commonly found in a spare parts box. Experimental results suggest a reduction of detector diode losses, after an improvement of the receiver's tuning characteristics. All tested diodes gave broad tuning responses prior to utilization of feedback, selectivity improving noticeably thereafter.

Fig.1 shows the schematic diagram of the modified crystal receiver employed by the author. A passive feedback loop can be observed to exist in the circuit, that which comprises L3. This coil, wound over the cold end of L2, acts as a sort of tickler coil, feeding back to the tuning tank RF-detected currents of the same frequency and phase as those induced by the incoming wave. An equivalent circuit for carrier frequencies is shown in Fig.2, where vA(t) and vK(t) are the diode's anode and cathode potentials to ground, respectively.

Shockley's diode equation states:

$$i_D = I_S\left(e^{\frac{v_D}{nV_T}} - 1\right) \quad \ldots(1)$$

where iD is the diode's current in amperes, vD is the diode's anode-cathode voltage drop in volts, IS is the inverse saturation current in amperes, n is the diode's ideality factor and VT is 0.026 Volts. IS is a scale factor, such that if we wish iD in microamperes, for example, we need only put IS in these same units.

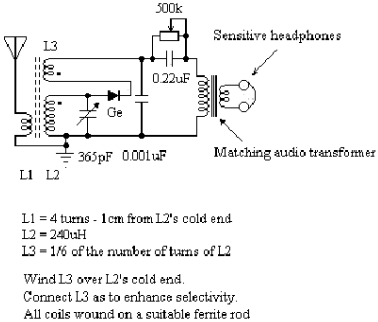


Fig.1 Crystal set with enhanced selectivity

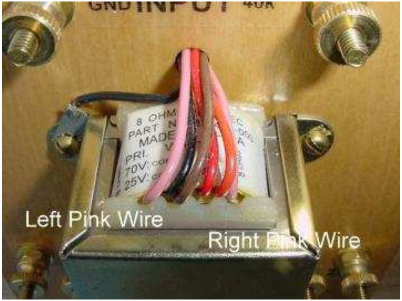
Eq.(1) may be expanded into a Taylor's series:

$$i_D = A_0 + A_1 v_D + A_2 v_D^{\ 2} + A_3 v_D^{\ 3} + \ldots \quad \ldots(2)$$

where

$$v_D = v_A(t) - v_K(t) \quad \ldots(3)$$

and A0 , A1 , A2 , A3 , … are constants having the values:

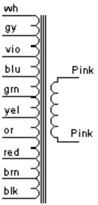


All values are relative to the left side pink tap.

Color	Impedance @ 300Hz
White	40k ohms
Gray	20k ohms
Violet	10k ohms
Blue	5k ohms
Green	2.5k ohms
Yellow	1.5k ohms
Orange	900 ohms
Red	500 ohms
Brown	250 ohms
Pink (R)	8 ohms

All values are relative to the black (blk) tap.

Color	Resistance	Inductance	XL @ 300 hz	Rounded Value
White (WH)	1424.3 ohms	24 H	45.239k ohms	40k ohms
Gray (GRY)	886.4 ohms	12.04 H	22.694k ohms	20k ohms
Violet (VIO)	516.5 ohms	6.06 H	11.423k ohms	10k ohms
Blue (BLU)	260.1 ohms	3.04 H	5.730k ohms	5k ohms
Green (GRN)	81.8 ohms	1.565 H	2.950k ohms	2.5k ohms
Yellow (YEL)	56 ohms	787 mH	1.483k ohms	1.2k ohms
Orange (OR)	38.2 ohms	398 mH	750.2 ohms	600 ohms
Red (RED)	26 ohms	197 mH	371.3 ohms	300 ohms
Brown (BRN)	18.2 ohms	98 mH	184.7 ohms	150 ohms
Pink to Pink	0.5 ohms	5.23 mH	9.86 ohms	8 ohms



$$A_0 = 0$$

$$A_1 = \frac{I_S}{nV_T}$$

$$A_2 = \frac{I_S}{(nV_T)^2} \cdot \frac{1}{2!}$$

$$A_3 = \frac{I_S}{(nV_T)^3} \cdot \frac{1}{3!}$$

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Coefficient A1 is responsible for the tuning tank's RF loading at the carrier's frequency and is equal to the diode's conductance at vD = 0, or zero-crossing conductance. On the other hand, coefficient A2 is responsible for the square-law AM demodulation of the incoming RF signal.

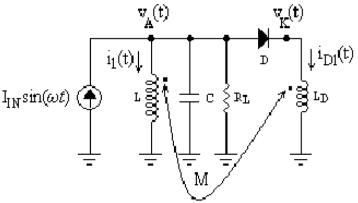


Fig.2 Equivalent circuit at carrier frequencies

In Fig.2, IIN sin(ωt) is the RF signal current induced by the passing radio wave and driving the tuning tank. Resistor RL represents the tuned circuit's parallel RF losses. M is the mutual inductance existing between coils L and LD (L and LD represent L2 and L3, respectively). We are interested in the fundamental frequency component of the diode's current, i.e., the carrier-frequency component iD1(t). Thus, we may write:

$$i_{D1}(t) = A_1 v_D = A_1[v_A(t) - v_K(t)] \quad \ldots(4)$$

In the frequency domain:

$$I_{D1}(j\omega) = A_1 V_A(j\omega) - A_1 V_K(j\omega) \quad \ldots(5)$$

From Fig.2:

$$I_{IN}(j\omega) = I_1(j\omega) + V_A(j\omega)\cdot\left(j\omega C + \frac{1}{R_L}\right) + I_{D1}(j\omega) \quad \ldots(6)$$

$$V_A(j\omega) = I_1(j\omega)\cdot j\omega L + I_{D1}(j\omega)\cdot j\omega M \quad \ldots(7)$$

and:

$$V_K(j\omega) = I_{D1}(j\omega)\cdot j\omega L_D + I_1(j\omega)\cdot j\omega M \quad \ldots(8)$$

Substituting Eqs.(7) and (8) in (5):

$$I_{D1}(j\omega) = A_1 I_1(j\omega)\cdot j\omega L + A_1 I_{D1}(j\omega)\cdot j\omega M - A_1 I_{D1}(j\omega)\cdot j\omega L_D - A_1 I_1(j\omega)\cdot j\omega M$$

$$= A_1 I_1(j\omega)\cdot j\omega(L - M) + A_1 I_{D1}(j\omega)\cdot j\omega(M - L_D)$$

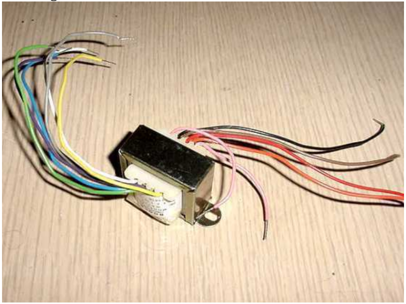
Then:

$$I_{D1}(j\omega)\cdot[1 - A_1 j\omega(M - L_D)] = A_1 I_1(j\omega)\cdot j\omega(L - M)$$

Impedance transformation ratios are then N2 = LT / L1 for each tap. The impedance ratio at 300 Hz or whatever frequency in the autotransformer´s pass band is approximately equal (for k ~1) to the inductance ratio. Each tap is recognized by its impedance. I find it useful to calculate things at 300Hz and then scale the figures to the desired frequency. If it is desired 40k ohms to be the maximum impedance point, then it is a simple matter of calculating 40*L1 / LT in kohms to find the corresponding impedance for the selected tap.

This is the methodology for determining the corresponding impedances assigned to each tap in the Bogen T-725 autotransformer (and for any other useful autotransformer for audio applications).

The Bogen T-725 Autotransformer



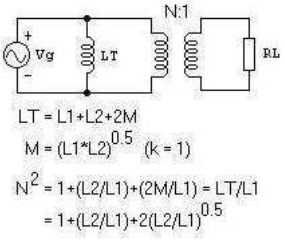
Bogen Specs.

$$\frac{I_2}{V_g} = \frac{R_L + j\omega L_1}{j\omega(L_1 + L_2 + 2M)R_L}$$

$$= \frac{1}{j\omega(L_1 + L_2 + 2M)} + \frac{L_1}{(L_1 + L_2 + 2M)R_L}$$

$$= \frac{1}{j\omega(L_1 + L_2 + 2M)} + \frac{1}{\left(1 + \frac{L_2}{L_1} + \frac{2M}{L_1}\right)R_L}$$

which suggests the following transformer-like network equivalent:



Transformer-like network equivalent of the autotransformer loaded by a resistive load RL.

LT = L1+L2+2M is the measured inductance of the whole winding and L1 is the inductance measured from the tap to the lower end of the winding.

and

$$I_{D1}(j\omega) = \frac{A_1 I_1(j\omega)\cdot j\omega(L - M)}{1 - A_1 j\omega(M - L_D)} \quad \ldots(9)$$

Substituting for ID1(jω) in Eq.(7):

$$V_A(j\omega) = I_1(j\omega)\cdot j\omega L + \frac{A_1 I_1(j\omega)\cdot j\omega(L - M)j\omega M}{1 - A_1 j\omega(M - L_D)}$$

$$= I_1(j\omega)\cdot j\omega L - \frac{A_1 I_1(j\omega)\cdot \omega^2(L - M)M}{1 - A_1 j\omega(M - L_D)}$$

$$= I_1(j\omega)\cdot \frac{A_1\omega^2(M^2 - LL_D) + j\omega L}{1 - A_1 j\omega(M - L_D)}$$

$$= I_1(j\omega)\cdot \frac{A_1\omega^2 LL_D(k^2 - 1) + j\omega L}{1 - A_1 j\omega(M - L_D)}$$

where $M^2 = k^2 LL_D$ and k = coupling coefficient between L and LD.

Then:

$$I_1(j\omega) = V_A(j\omega)\cdot \frac{[1 - A_1 j\omega(M - L_D)]}{A_1\omega^2 LL_D(k^2 - 1) + j\omega L} \quad \ldots(10)$$

Substituting in Eq.(9):

$$I_{D1}(j\omega) = V_A(j\omega)\cdot\frac{A_1 j\omega(L-M)}{A_1\omega^2 LL_D(k^2-1)+j\omega L}$$ …(11)

If $A_1\omega L_D(1-k^2) << 1$ then:

$$I_1(j\omega) = V_A(j\omega)\cdot\frac{[1-A_1 j\omega(M-L_D)]}{j\omega L}$$ …(12)

Eq.(11) reduces to:

$$I_{D1}(j\omega) = V_A(j\omega)\cdot A_1\left(1-\frac{M}{L}\right)$$ …(13)

Comparison with Eq.(5) yields:

$$V_K(j\omega) = V_A(j\omega)\cdot\frac{M}{L}$$ …(14)

Substituting Eqs.(12) and (13) in Eq.(6):

$$I_{IN}(j\omega) = V_A(j\omega)\cdot\frac{[1-A_1 j\omega(M-L_D)]}{j\omega L}+V_A(j\omega)\cdot\left(j\omega C+\frac{1}{R_L}\right)+V_A(j\omega)\cdot A_1\left(1-\frac{M}{L}\right)$$

$$= V_A(j\omega)\cdot\frac{(1-\omega^2 LC)+j\omega L\left[A_1\left(1-\frac{M}{L}\right)+\frac{1}{R_L}-\frac{A_1(M-L_D)}{L}\right]}{j\omega L}$$

At resonance:

$$1-\omega^2 LC = 0$$

which yields:

$$(R_L + j\omega L_1)I_1 = j\omega(L_1+M)I_2$$

Then,

$$I_1 = \frac{j\omega(L_1+M)}{R_L+j\omega L_1}I_2$$ ...(3)

Eq. (1) may be arranged as:

$$V_g = j\omega I_2(L_1+L_2+2M) - j\omega I_1(L_1+M)$$

Substituting eq. (3) into the above expression:

$$V_g = j\omega I_2(L_1+L_2+2M)+\frac{\omega^2(L_1+M)^2}{R_L+j\omega L_1}I_2$$

$$= \frac{j\omega I_2(L_1+L_2+2M)R_L - \omega^2 L_1 I_2(L_1+L_2+2M)+\omega^2(L_1+M)^2 I_2}{R_L+j\omega L_1}$$

$$= \frac{j\omega I_2(L_1+L_2+2M)R_L + \omega^2 I_2(M^2-L_1L_2)}{R_L+j\omega L_1}$$

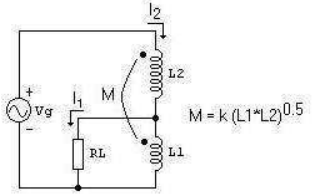
The mutual inductance M of L1 and L2 is given by M = k (L1L2)0.5, where k is the coupling coefficient. For values of k very close to unity, M2-L1L2 nearly vanishes and :

$$V_g = \frac{j\omega I_2(L_1+L_2+2M)R_L}{R_L+j\omega L_1}$$

The input admittance of the autotransformer network is computed as:

Useful Calculations for better understanding the use of The Bogen T-725 Autotransformer
By Ramon Vargas Patron
http://www.inictel-uni.edu.pe/index.php?option=com_content&view=article&id=235&Itemid=152

Consider a lossless autotransformer driven by a voltage source Vg supplying power to a resistor RL. See figure below.



Lossless autotransformer with resistive load

Mesh equations describing the above network can be writtten as:

$$V_g = j\omega(L_2 + M)I_2 + j\omega(L_1 + M)(I_2 - I_1) \quad ...(1)$$

$$I_1 = \frac{1}{R_L}\left[j\omega L_1(I_2 - I_1) + j\omega M I_2\right] \quad ...(2)$$

From the above equation:

$$\omega_R = \frac{1}{\sqrt{LC}} \quad ...(15)$$

The tune circuit's admittance at ωR is:

$$Y(j\omega_R) = \frac{I_{IN}(j\omega_R)}{V_A(j\omega_R)} = A_1\left(1 - \frac{M}{L}\right) + \frac{1}{R_L} - A_1\left(\frac{M - L_D}{L}\right)$$

$$= A_1\left(1 - 2\frac{M}{L} + \frac{L_D}{L}\right) + \frac{1}{R_L}$$

$$= A_1\left(1 - 2k\sqrt{\frac{L_D}{L}} + \frac{L_D}{L}\right) + \frac{1}{R_L}$$

If k ≈ 1:

$$Y(j\omega_R) = G(\omega_R) \approx A_1\left(1 - \sqrt{\frac{L_D}{L}}\right)^2 + \frac{1}{R_L} \quad ...(16)$$

The diode's conductance at resonance is then:

$$G_D = A_1\left(1 - \sqrt{\frac{L_D}{L}}\right)^2 \quad ...(17)$$

A smaller GD means reduced parallel losses in the tuned circuit due to the diode's zerocrossing
resistance value.

We conclude that passive positive feedback in the crystal receiver acts reducing

detector diode losses by a factor $\left(1-\sqrt{\frac{L_D}{L}}\right)^2$ and, within the range of frequencies for
which the equivalent model is valid, it won´t change the tuned circuit's resonance
radian frequency $\omega_R = \frac{1}{\sqrt{LC}}$.

References

[1] Tongue, Ben. Crystal Radio Set Systems: Design, Measurement and Improvement--
Practical considerations, helpful definitions of terms and useful explanations of some concepts used in this site.
http://www.bentongue.com/xtalset/0def_exp/0def_exp.html
(seen on May 10th 2009)

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Optimization of the LD/L Ratio of a Selectivity-Enhanced Germanium-Detector Crystal Radio

By Ramon Vargas Patron

http://www.inictel-uni.edu.pe/index.php?option=com_content&view=article&id=235&Itemid=152

Our article "On the Reduction of Detector Diode Losses in a Crystal Radio" described a simple feedback technique for reducing the zero-crossing conductance of a germanium crystal diode. The present report will show how to maximize the signal voltage across the diode at resonance while still maintaining improved selectivity. This will also give maximum detected audio output from the receiver.

Under the assumption the aforementioned article showed that:

$$I_{IN}(j\omega_R) = V_A(j\omega_R)\cdot\left[A_1\left(1-\sqrt{\frac{L_D}{L}}\right)^2 + \frac{1}{R_L}\right] \quad \ldots(1)$$

where ωR = 2πfR is the radian resonant frequency of the tuning tank.

The diode's anode-cathode voltage drop is:

$$V_A(j\omega_R) - V_K(j\omega_R) = V_A(j\omega_R)\cdot\left(1-\frac{M}{L}\right)$$

$$= V_A(j\omega_R)\cdot\left(1-\sqrt{\frac{L_D}{L}}\right)$$

assuming a coupling coefficient k ≈ 1.

transformer. Accordingly, the optimum values for Rg and RL should be:

$$R_g = \frac{447}{20} = 22.35kohms$$

$$R_L = 22.35\frac{X_S}{X_P} = 0.954kohms$$

Technical data was found recently on this transformer describing it as a Philmore 20k:1k 50mw input transformer (no more info available).

Case #2

Our second real-world example deals with the Calrad 45-700 audio transformer, specified by the manufacturer as a 100k:1k matching device. Measurements were taken to verify technical data found on the Internet. Results are tabulated below.

Winding	Wire Color Code	Inductance	Copper Losses
Primary	Green-Red	Lp = 52.5H	Rp = 2.11k ohms
Secondary	Green-White	Ls = 0.55H	Rs = 69.6 ohms

Calculated reactances at 300Hz are Xp = 98.96k ohms and Xs = 1.036k ohms for the primary and secondary, respectively, very close to the rated transformed and load impedances (100k ohms and 1k ohms).

The above values suggest that, under rated loading and matched conditions, the Calrad 45-700 will yield a lower corner frequency approximately equal to 150Hz.

Acknowledgements

The author would like to express here his gratefulness to Steven Coles, Gil Stacy, Ben Tongue and Dave Schmarder for their most kind technical support and encouragement.

From eq.(7) we already know that at f3dB the primary's reactance equals Rg/2. At two times f3dB, the reactance will equal Rg. This is a useful result, stating that at the lower end of the flat passband the primary's reactance will be equal to Rg.

We can use the above results in the following way. From the formula for a coil's reactance:

$$X_L = 2\pi f L \quad \ldots(9)$$

the impedance of the primary winding at 300Hz (neglecting resistive losses) is found to be Xp = 36.757k ohms. The secondary winding yields a value Xs = 1.57k ohms. Accordingly, Rg should be 36.757k ohms for a –3dB frequency of 150Hz, the optimum load being RL = 1.57k ohms.

A hearing device having an effective average audio impedance around 1.5k ohms will be matched to a detector diode's output audio impedance of 30…..40k ohms. This is likely a value for a typical germanium 1N34 diode in an average performance crystal set using a tapped detector coil. If higher load impedances are used, the –3dB frequency will be shifted upwards and there will be losses at bass frequencies.

For core losses to be neglected, the transformed load impedance should satisfy the following relationship:

$$R_L' = N^2 R_L \approx \frac{L_p}{L_s} R_L = \frac{X_p}{X_s} R_L \ll R_c$$

In the present case, Rc was found to be 447k ohms (the method for taking this measurement will be discussed in a future article). As a gross approximation, a value for Rc equal to 20 times N2RL nominal may be assumed for a good audio

Substituting Eq.(1) into the expression for the diode's voltage drop we obtain:

$$V_A(j\omega_R) - V_K(j\omega_R) = I_{IN}(j\omega_R)\cdot \frac{\left(1-\sqrt{\frac{L_D}{L}}\right)}{\left[A_1\left(1-\sqrt{\frac{L_D}{L}}\right)^2 + \frac{1}{R_L}\right]}$$

$$= I_{IN}(j\omega_R)\cdot R(x) \quad \ldots(2)$$

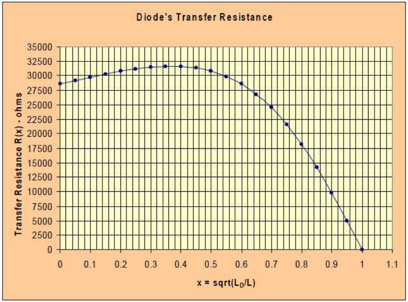
where $x = \sqrt{\frac{L_D}{L}}$. We call R(x) the detector diode's transfer resistance at carrier frequencies.

Maximizing expression (2) will yield stronger detected RF currents. Therefore, optimization of the LD/L ratio of the receiver proves itself a necessary task.

A typical graph for R(x) is shown in Fig.1, where sample values for A1 and RL have been chosen as ohms and RL = 100k ohms. The independent variable $x = \sqrt{\frac{L_D}{L}}$ takes values between zero and unity. A maximum in the graph may be seen to occur at:

$$x = x_M = 1 - \frac{1}{\sqrt{A_1 R_L}} \quad \ldots(3)$$

that is, at xM = 0.3675.



Diode's Transfer Resistance

Fig.1 Typical diode transfer resistance graph when $A_1 = \frac{1}{40}$ and RL= 100kohm.

This is exactly the value of x for which the modified zero-crossing conductance of the diode equals the conductance of the tuned circuit at resonance, according to the maximum power transfer principle, i.e., :

$$A_1(1-x)^2 = \frac{1}{R_L} \quad \ldots(4)$$

technical info was available at that time. The only known fact was that the external connection to the windings was a set of flexible green, red, white and black wires.

With the help of an audio generator and an oscilloscope some basic measurements were made. First, the green-red wires were identified as corresponding to the high-impedance winding (primary) and the white-black pair as that pertaining to the low-impedance side (secondary). Then, an approximate value for the turns ratio N was obtained. A 0.1V peak-amplitude 1kHz signal was applied to the primary, giving 0.022V peak across the unloaded secondary. This yielded a 4.54:1 voltage transformation ratio or N. However, it is recommended the turns ratio be measured under rated-load conditions (impractical at this point of our work, as we knew nothing about the impedances of this little transformer).

The author could also get a hand on a B&K 875A LCR meter for inductance and resistance measurements. The high-impedance winding measured LP = 19.5H and DC resistive losses of Rp = 1.236k ohms. The low-impedance side showed LS = 0.833H and DC resistive losses of Rs = 153 ohms. Applying eq.(8) a value of 4.84 for N was obtained. This value is believed to be a better approximation for N.

As mentioned before, the minimum acceptable bandwidth should be 300Hz to 3000Hz, flat. In practice, amplitude response variations of +/-1dB relative to the midband are acceptable. For the response at 300Hz to be within this tolerance, we must select 150Hz as the –3dB frequency (at two times the corner frequency, the response is within 1dB of the value found at mid frequencies).

The above equation tells us that, under matched conditions, at the lower –3dB frequency the reactance of the magnetizing inductance equals one half of the source resistance.

Then:

$$\omega_{3dB} = \frac{R_g}{2L_m}$$

and:

$$f_{3dB} = \frac{1}{2\pi} \cdot \frac{R_g}{2L_m} \quad \ldots(7)$$

If the shunt inductance Lm and the source resistance Rg are known quantities, f3dB can be readily obtained. On the other hand, if Lm and the turns ratio N are known, selecting f3dB will yield the corresponding values for Rg and the optimum load RL, recalling that Rg = N2RL.

A useful approximation for N is:

$$N = \sqrt{\frac{L_p}{L_s}} \quad \ldots(8)$$

which requires that LP and LS be known. Also, being k ≈ 1, we may write Lm ≈ LP.

Working out some examples

Case # 1

Some months back the author received a small audio transformer having the code ST-11 stamped on its side. No

$$x_M = 1 - \frac{1}{\sqrt{A_1 R_L}}$$

$$= \sqrt{\frac{L_D}{L}}_{OPTIMUM}$$

The maximum of the curve in Fig.1 is given by:

$$R(x_M) = \frac{(1 - x_M)}{A_1(1 - x_M)^2 + \frac{1}{R_L}}$$

$$= \frac{\frac{1}{\sqrt{A_1 R_L}}}{A_1\left(\frac{1}{A_1 R_L}\right) + \frac{1}{R_L}} \quad \ldots(5)$$

$$= \frac{1}{2}\sqrt{\frac{R_L}{A_1}}$$

For the said sample values of A1 and RL, R(xM) = 31.623k ohms. The net parallel resistance across the tuning tank at resonance is $R_{NET} = \frac{R_L}{2} = 50k\Omega$.

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Analysis of the Tuggle Front End in three parts:
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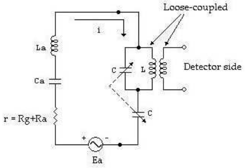
http://www.inictel-uni.edu.pe/index.php?option=com_content&view=article&id=235&Itemid=152

This article analyzes the Tuggle tuner, of common use in high-performance DX crystal sets. An equivalent circuit for the antenna-ground system with the tuner connected is shown in Fig. 1 below. It must be recognized that there is some stray capacitance of the rotor and frame of the two-gang variable capacitor to ground. This should be shown as a fixed capacitor across the bottom variable capacitor. Its presence will reduce the maximum frequency to which the circuit will tune. However, in the present analysis this stray capacitance is neglected.



-Ea is the voltage induced in the antenna by the radio wave.
-La is the equivalent series inductance of the antenna.
-Ca is the equivalent series capacitance of the antenna.
-r accounts for the antenna radiation resistance Ra and ground losses Rg.

Fig. 1 The Tuggle front end connected to an antenna-ground system

The mesh current is described by:

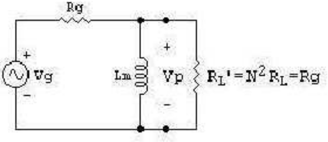


Fig.7 Equivalent circuit for calculation of the -3dB frequency

Analysis of the circuit yields for the primary's voltage:

$$V_p = \frac{V_g}{2} \cdot \frac{j\omega L_m}{\frac{R_g}{2} + j\omega L_m}$$

where ω = 2πf is the radian frequency and $j = \sqrt{-1}$.

Then, for the half-power point [eq.(5)]:

$$\frac{V_g}{2\sqrt{2}} = \frac{V_g}{2} \cdot \frac{\omega L_m}{\sqrt{\frac{R_g^2}{4} + \omega^2 L_m^2}}$$

or:

$$\frac{1}{2} = \frac{\omega^2 L_m^2}{\frac{R_g^2}{4} + \omega^2 L_m^2}$$

Simplifying:

$$\frac{R_g}{2} = \omega L_m \qquad \ldots(6)$$

If the device has been manufactured for audio frequency operation (there are reports saying that small 60Hz low-voltage power-line transformers have been successfully used as matching devices), then chances are that it will reproduce frequencies up to at least 3000Hz. However, attenuation at lower frequencies will be strongly dependent on the magnetizing inductance Lm and the value of the source resistance Rg (high-quality units may reproduce frequencies down to 50Hz +/-1dB, referenced to the midband).

The equivalent circuit shown in Fig.6.c is very valuable for evaluating transformer operation at frequencies below the midband. We will use this model for calculation of fL, the lower -3dB frequency (please see Fig.5). At this frequency, the power delivered to the load RL will be one half of that available in the midband, this is, it will be 3dB down.

Computing the half-power point

If we lower the operating frequency, eventually the primary's magnetizing inductance Lm will start shunting the available signal, reducing the output power. Recalling that at mid frequencies Vp = Vg/2, the half-power point of the response will be described by that frequency at which the amplitude of the primary's voltage falls to:

$$V_p = \frac{V_g}{2\sqrt{2}} \quad \ldots(5)$$

With Fig.6.c in mind, we may draw the circuit of Fig.7 to help us in the calculation of the lower -3dB frequency.

$$\ldots(1)$$

$$I = \frac{Ea}{r + j\left(\omega La - \frac{1}{\omega Ca}\right) + \overline{Z} - \frac{j}{\omega C}}$$

$$\overline{Z} = \frac{1}{\overline{Y}}$$

where:

$$\overline{Y} = \frac{1}{j\omega L} + j\omega C$$

$$= j\left(\omega C - \frac{1}{\omega L}\right)$$

$$= j\left(\frac{\omega^2 LC - 1}{\omega L}\right)$$

$$\therefore \quad \overline{Z} = \frac{1}{j}\left(\frac{\omega L}{\omega^2 LC - 1}\right)$$

or:

$$\overline{Z} = j\left(\frac{\omega L}{1 - \omega^2 LC}\right)$$

Then, $I = I_{MAX}$ when:

$$\left(\omega La - \frac{1}{\omega Ca}\right) + \left(\frac{\omega L}{1-\omega^2 LC}\right) - \frac{1}{\omega C} = 0$$

This is, when:

…(2)

$$\omega\left(La + \frac{L}{1-\omega^2 LC}\right) - \frac{1}{\omega\left(\frac{CaC}{Ca+C}\right)} = 0$$

which is satisfied at certain radian frequency ω_r.
At this frequency, the L-C tank circuit behaves as an equivalent inductance

$$\frac{L}{1-\omega_r^{\,2} LC}$$

Usually, L is much greater than L_a for antennas used in crystal set work. Then,

$$La << \frac{L}{1-\omega_r^{\,2} LC}$$

Equation (2) can be written as:

$$\frac{\omega_r L}{1-\omega_r^{\,2} LC} - \frac{1}{\omega_r\left(\frac{CaC}{Ca+C}\right)} = 0$$

We can then write:

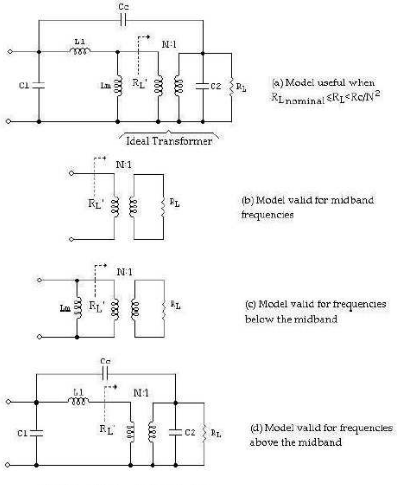


Fig.6 Simplified equivalent circuits for the audio transformer

We are interested in knowing if a specific transformer will efficiently match the given impedance levels Rg and RL. We would also like to know if the 300Hz~3000Hz bandwidth (minimum) will be accomplished by using this audio transformer.

$R_c \rangle\rangle R_L{}' = R_g$

$k \approx 1$

Also, at rated loads, the effects of C1, C2 and Cc should be noticeable only at higher frequencies, beyond the midband.

A typical amplitude versus frequency response curve for an audio transformer is shown in Fig.5. In this figure, 0dB refers to the output level at midband frequencies. At frequencies fL and fH the output is down by 3dB.

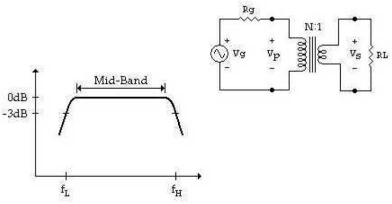


Fig.5 Typical amplitude response of an audio transformer

For a load resistance RL equal to or greater than the rated value RL nominal , but much smaller than Rc/N2, the transformer may be represented by the simplified models of Fig.6.

$$\frac{\omega_r L}{1-\omega_r{}^2 LC} = \frac{Ca + C}{\omega_r CaC}$$

After some algebraic manipulation we obtain:

$$\omega_r{}^2 LC\left(\frac{2Ca + C}{Ca + C}\right) = 1 \qquad \ldots(3)$$

The equivalent capacitance resonating with L is:

$$C_{eq} = C\left(\frac{2Ca + C}{Ca + C}\right)$$

Clearly, C_{eq}>C.

Following is a numerical example illustrating the use of the above results.
Let C be a variable capacitance with C_{MIN} = 20 pF and C_{MAX} = 475 pF. Let also C_a be
200 pF. Then, C_{eq} varies between C_{eqMIN} = 38.18 pF and C_{eqMAX} = 615.74 pF.

If we wish to tune the MW broadcast band starting at 530 kHz, then the required inductance L will be:

$$L = \frac{1}{\omega_{rMIN}{}^2 C_{eqMAX}}$$

$$= 146.45uH$$

The circuit will tune up to:

$$f_{MAX} = f_{MIN}\left(\frac{C_{eqMAX}}{C_{eqMIN}}\right)^{\frac{1}{2}}$$

$$= 2.128MHz$$

If we use for C a variable capacitance with C_{MIN} = 20 pF and C_{MAX} = 365 pF, then
C_{eqMIN} = 38.18 pF and C_{eqMAX} = 494.20 pF, giving for the required inductance L a value of 182.46 uH. The circuit will tune up to f_{MAX} = 1.906 MHz.

Acknowledgements: Special thanks are given to Ben Tongue for his comments on the manuscript and for encouraging further mathematical analysis of the circuit regarding bandwidth variation with frequency, which will be done shortly.

• L1 is the equivalent leakage inductance referred to the primary side. It results from magnetic flux not mutually linked by the windings and contributes to losses at high frequencies.

• N is the “turns ratio”.

• The copper loss (resistance) of the primary and secondary windings is represented by Rp and Rs, respectively.

• Rc represents core losses. Contributions to these losses come from eddy-currents and hysteresis behaviour.

• C1 and C2 are the intra-winding capacitances and Cc is the inter-winding capacitance. These three are stray (parasitic) capacitances. They also contribute to power losses at the high-frequency end of the response.

• If the windings are DC-isolated from each other, the short connecting the lower ends of the ideal transformer in the model should be substituted by a second capacitor Cc connected between the lower input and output terminals.

Selecting a suitable transformer

For crystal set use, a good transformer should have a flat response from 300Hz to 3000Hz (or better) when loaded following manufacturer´s specs. Accordingly, the following relationships should be satisfied:

$$R_g \rangle\rangle R_p$$

$$R_L \rangle\rangle R_s \qquad \ldots(4)$$

Modeling a transformer

An equivalent network for an audio transformer can be seen in Fig.4. Here, circuit parameters have been defined in terms of the inductances LP and LS of the primary and secondary windings, the coupling coefficient k between these windings, stray capacitances and losses.

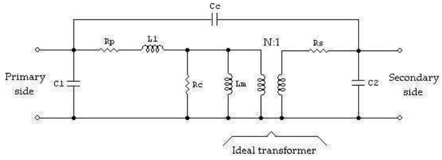


Fig.4 Equivalent circuit for an audio transformer showing components that tailor the frequency response

The following relationships apply:

$$L_m = k^2 L_p \quad \ldots(3.1)$$

$$L_1 = (1 - k^2) L_p \quad \ldots(3.2)$$

$$N = k\sqrt{\frac{L_p}{L_s}} \quad \ldots(3.3)$$

• Lm is the magnetizing (or shunt) inductance. Its finite value is responsible for power losses at low audio frequencies.

Analysis of the Tuggle Front End – Part II

We shall now consider the Tuggle tuner delivering power to a load. First, we must account for the parallel RF losses of the unloaded tuned circuits. Let R_P represent the losses of the tank circuit comprising L_1 and C, and R_{TNK} those of L_2 and C_2 (please see Fig. 1).

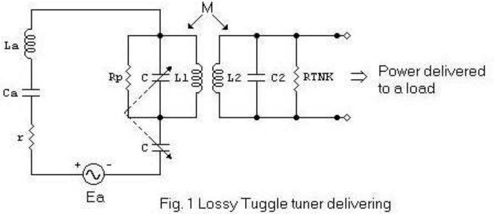


Fig. 1 Lossy Tuggle tuner delivering power to the secondary load

The load consists of a diode detector D in series with an audio load R_L (usually an audio transformer matching a pair of 2k ohms DC resistance magnetic headphones or low-impedance sound powered phones to the detector) and is coupled to the tuner via the magnetic coupling existing between L_1 and L_2, being M the mutual inductance of the coils. The secondary is tuned to the same radian frequency as the primary. An schematic for the load can be seen in Fig. 2.

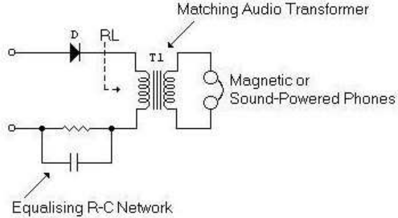


Fig. 2 Load coupled to the Tuggle tuner

Usually, it is assumed that optimum RF power transfer occurs when the antenna-ground system resonance resistance r is matched to the unloaded-secondary resonance parallel resistance R_{TNK}, with the diode detector´s input resistance matched to this combination. Thus,

$$\frac{1}{R_{OPT}} = \frac{1}{R_{TNK}} + \frac{1}{R_{TNK}} + \frac{1}{\left(\frac{R_{TNK}}{2}\right)}$$

or

$$R_{OPT} = \frac{R_{TNK}}{4}.$$

Next, consider the situation where RL differs from Rg and we still want the maximum available power delivered to RL. At audio frequencies, it is common practice to connect a matching transformer between the source and the load. This device permits transformation of impedance levels, such that the generator "sees" an equivalent load RL' = Rg. Maximum power is then available and it will be transferred to RL. Please refer to Fig.3.

The transformer should be a low-loss type, in order to restrict power losses to a minimum. Usually, a specially treated steel-laminated core is used when higher permeability values and very tight magnetic coupling between windings is required.

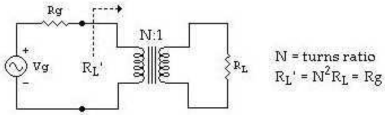


Fig.3 Audio transformer coupling the load R_L to the voltage source

In a crystal receiver, Rg represents the detector diode's output resistance at audio frequencies and RL, the effective average impedance of a pair of 2k ohms DC resistance magnetic headphones, sound powered headphones or piezoelectric ceramic or crystal earpiece.

RL must be matched to Rg for maximum power transfer to the hearing device.

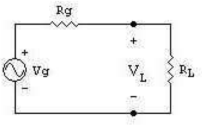


Fig.2 Generator delivering power to a resistive load R_L

The average power dissipated by RL is:

$$P_L = \frac{V_L^2}{2R_L} \quad \dots(1)$$

where VL is the peak value of the voltage across the load. VL is computed as:

$$V_L = V_g \frac{R_L}{R_L + R_g}$$

Substitution into eq.(1) yields:

$$P_L = V_g^2 \frac{R_L}{2(R_L + R_g)^2}$$

Maximum power is delivered to the load when RL = Rg. In this particular case:

$$P_L = P_{LMAX} = \frac{V_g^2}{8R_g} \quad \dots(2)$$

This is the maximum available power.

This is the overall parallel RF resistance of the secondary tank under matched conditions, and suggests that the unloaded Q of this tank circuit has been reduced to ¼ of its value.
In this case, the net parallel resistance to be coupled to the primary is:

$$R_2 = \frac{R_{TNK}\left(\frac{R_{TNK}}{2}\right)}{R_{TNK} + \left(\frac{R_{TNK}}{2}\right)}$$

$$= \frac{R_{TNK}}{3} \quad \dots(1)$$

We shall work on this later.

Some circuit equivalents

In Fig.1, let´s replace L_1 and the coupled secondary circuit by the equivalent shown in Fig.3.a, which in turn can be replaced by the transformer circuit shown in Fig.3.b.

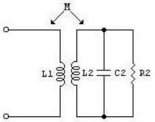


Fig.3.a Magnetically coupled circuits

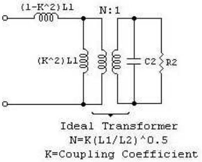


Fig.3.b Equivalent transformer circuit

In the transformer circuit, the impedance coupled to the primary side consists of a capacitance C_2 / N^2 in parallel with a resistance N^2R_2. Both are in parallel with the magnetizing inductance K^2L_1 (please see Fig.4.a).

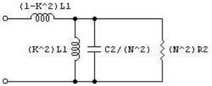


Fig.4.a Equivalent circuit as seen from the primary side

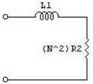


Fig.4.b Reduced equivalent at $\omega = \omega_r$

K^2L_1 and C_2 / N^2 resonate at a frequency

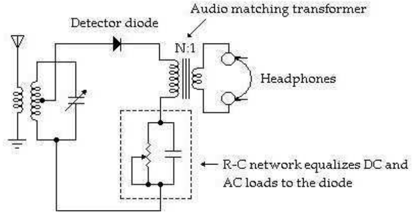


Fig.1 Basic xtal set with headphones impedance-matched to the detector

This article will present technical material the author believes could be helpful to the hobbyist when selecting a suitable audio transformer for his set or when studying utilization of the one just found in the spare parts box.

We shall begin making some basic power calculations. First, consider a sine-wave generator delivering power to a resistive load RL, as shown in Fig.2. Vg is the peak amplitude of the source voltage and Rg is the source resistance.

Optimal Loading of Audio Transformers for Crystal Set Use
By Ramon Vargas Patron
http://www.inictel-uni.edu.pe/index.php?option=com_content&view=article&id=235&Itemid=152

Crystal receivers have attracted for decades the attention of radio enthusiasts, mainly because of its low parts count and capability for long-distance reception when connected to an efficient antenna-ground system.

Overall crystal set design (and construction) criteria try to get the most of the RF power intercepted by the antenna into the headphones, ultimately as audio-frequency power, this power being converted into sound by the hearing element.

In recent years, utmost importance has been given to the utilization of matching audio transformers and suitable hearing devices for ultimate volume improvements in these receivers. Fig.1 shows the schematic diagram of a basic crystal set featuring an audio impedance-matching stage.

$$\omega_0 = \frac{1}{\sqrt{K^2 L_1 \left(\frac{C_2}{N^2}\right)}}$$

$$= \frac{1}{\sqrt{L_2 C_2}} = \omega_r$$

as

$$N^2 = K^2 \frac{L_1}{L_2}.$$

The equivalent circuit of Fig.4.a reduces to that of Fig.4.b, taking into account that for crystal set use, normally K<<1.
Up to this point, in the tuner side we have the equivalent circuit depicted in Fig.5.

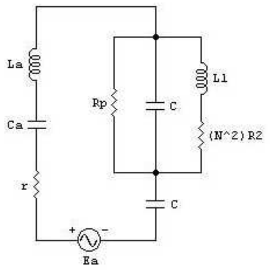


Fig.5 Equivalent circuit of the tuner with coupled secondary load

The series-coupled resistance N^2R_2 can be transformed into a resistance R_{P1} in parallel with R_P. Using the known series-to-parallel "loss resistance" transformation we get:

$$R_{P1} = \frac{(\omega_r L_1)^2}{N^2 R_2} \qquad \ldots(2)$$

Let $Q_2 = \frac{R_2}{\omega_r L_2}$. Then:

$$BW = 2\Delta\omega = 2r\left(1+\frac{A}{R_P}\right)\left(\frac{1}{L_1\left(2+\frac{C}{Ca}\right)}\right)\left(\frac{Ca}{2Ca+C}\right) \quad \ldots(3)$$

in radians per second.

Next, we shall compute the bandwidths at three frequencies: 530kHz, 1MHz and 1.7MHz. Results are tabulated below.

Freq.(kHz)	BW (kHz)	Q	R_P (kΩ)	A (kΩ)	C (pF)	Ca (pF)	r (Ω)	L_1 (uH)
530	5.004	105.9	354.32	155.50	450	200	30	152
1000	13.47	74.24	558.7	190	100	200	30	152
1700	24.83	68.49	487.072	406.8	31	200	30	152

Results for BW are very close to those obtained from simulation of circuit of Fig.1.b.

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April 11th 2004

$$R_{P1} = \frac{\omega_r L_1}{K^2 Q_2} \quad \ldots(3)$$

Next, we compute the equivalent resistance R_T of the parallel combination of R_P and R_{P1}. It is given by:

$$R_T = \frac{R_P R_{P1}}{R_P + R_{P1}}$$

Substituting R_{P1} by its equivalent given by eq.(3):

$$R_T = \frac{R_P\left(\frac{\omega_r L_1}{K^2 Q_2}\right)}{R_P + \frac{\omega_r L_1}{K^2 Q_2}}$$

$$= \frac{R_P \omega_r L_1}{\omega_r L_1 + K^2 Q_2 R_P}$$

Letting $Q_1 = \frac{R_P}{\omega_r L_1}$ (unloaded Q of L_1-C tank) we obtain:

$$R_T = \frac{R_P}{1+K^2 Q_1 Q_2} \quad \ldots(4)$$

If $R_T >> \omega_r L_{eq} = \frac{\omega_r L_1}{1-\omega_r^2 L_1 C}$ (please refer to part I of this study) we can redraw the equivalent circuit of the tuner at resonance as indicated by Fig.6.a, and, by virtue of the above inequality, the resonant frequency ω_r will still be given by:

$$\omega_r^{\,2} L_1 C\left(\frac{2Ca+C}{Ca+C}\right)=1 \qquad \ldots(5)$$

Applying a parallel-to-series transformation, the equivalent circuit of Fig.6.b is obtained.

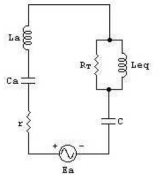

Fig.6.a Equivalent circuit of the tuner when $R_T \gg \omega_r L_{eq}$

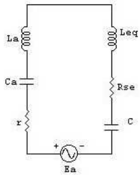

Fig.6.b Equivalent circuit after transformation

The series transformed resistance R_{se} is given by:

$$R_{se}=\frac{(\omega_r L_{eq})^2}{R_T}$$

$$\frac{2\omega_r(C_T+C)L_1\Delta\omega}{\left(\frac{Ca}{2Ca+C}\right)-2\omega_r L_1 C\Delta\omega}=\pm\omega_r C_T R_e$$

which simplifies to:

$$2L_1\left[C_T+C(1\pm\omega_r C_T R_e)\right]\Delta\omega=\pm C_T R_e\left(\frac{Ca}{2Ca+C}\right)$$

Now, if $\omega_r C_T R_e \ll 1$, then:

$$2\Delta\omega=\pm\frac{R_e}{L_1}\left(\frac{C_T}{C_T+C}\right)\left(\frac{Ca}{2Ca+C}\right)$$

this is:

$$\Delta\omega=\pm\frac{R_e}{2L_1}\left(\frac{Ca}{2Ca+C}\right)\left(\frac{Ca}{2Ca+C}\right)$$

$$=\pm\frac{R_{s1}}{L_{eq}}\left(\frac{Ca}{2Ca+C}\right) \qquad \ldots(2)$$

From part II of our study we can obtain the following expression for R_{s1}:

$$R_{s1}=r\left(1+\frac{A}{R_P}\right)$$

Then, the 3dB bandwidth is given by:

$$R_e^{\,2} + \left(\frac{\omega L_1}{1-\omega^2 L_1 C} - \frac{1}{\omega C_T} \right)^2 = 2R_e^{\,2}$$

or:

$$\frac{\omega L_1}{1-\omega^2 L_1 C} - \frac{1}{\omega C_T} = \pm R_e \qquad \ldots(1)$$

Let ω_r be the resonant frequency and $\omega = \omega_r + \Delta\omega$ the frequency at a –3dB point on the amplitude curve. The left hand member of eq. (1) can be written as:

$$\frac{\omega^2 L_1 (C_T + C) - 1}{(1-\omega^2 L_1 C)\omega C_T} = \frac{(\omega_r^{\,2} + 2\omega_r \Delta\omega)(C_T + C)L_1 - 1}{(1-(\omega_r^{\,2} + 2\omega_r \Delta\omega)L_1 C)\omega_r C_T}$$

with the following approximations:

$$(\omega_r + \Delta\omega)C_T \approx \omega_r C_T$$

$$(\omega_r + \Delta\omega)^2 = \omega_r^{\,2} + 2\omega_r \Delta\omega + \Delta\omega^2 \approx \omega_r^{\,2} + 2\omega_r \Delta\omega$$

Then:

$$\frac{\omega_r^{\,2} L_1 (C_T + C) + 2\omega_r (C_T + C) L_1 \Delta\omega - 1}{(1-\omega_r^{\,2} L_1 C - 2\omega_r L_1 C \Delta\omega)\omega_r C_T} = \pm R_e$$

or:

$$= (\omega_r L_{eq})^2 \left(\frac{1 + K^2 Q_1 Q_2}{R_P} \right)$$

$$= \frac{(\omega_r L_{eq})^2}{R_P} + (\omega_r L_{eq})^2 \frac{K^2 Q_1 Q_2}{R_P}$$

Let $R_s = \frac{(\omega_r L_{eq})^2}{R_P}$ and $R_{s1} = (\omega_r L_{eq})^2 \frac{K^2 Q_1 Q_2}{R_P}$.

R_s is the series term due to R_P and R_{s1}, that coming from the coupled resistance N^2R_2. Now, maximum RF power transfer to N^2R_2 occurs when:

$$r + R_s = R_{s1}$$

or when:

$$r + \frac{(\omega_r L_{eq})^2}{R_P} = (\omega_r L_{eq})^2 \frac{K^2 Q_1 Q_2}{R_P}$$

or

$$\frac{(\omega_r L_{eq})^2}{r} = \frac{R_P}{K^2 Q_1 Q_2 - 1} \qquad \ldots(6)$$

From part I of this study we know that:

$$L_{eq} = \frac{L_1}{1-\omega_r^{\,2} L_1 C}$$

$$= L_1\left(2+\frac{C}{Ca}\right) \quad \ldots(7)$$

being

$$\omega_r{}^2 = \frac{1}{L_1C}\left(\frac{Ca+C}{2Ca+C}\right)$$

Then:

$$\left(\omega_r L_{eq}\right)^2 = \frac{1}{L_1C}\left(\frac{Ca+C}{2Ca+C}\right)L_1{}^2\left(\frac{2Ca+C}{Ca}\right)^2$$

$$= \frac{L_1}{C}\left(2+\frac{C}{Ca}\right)\left(1+\frac{C}{Ca}\right)$$

Eq.(6) is now written as:

$$\frac{L_1}{rC}\left(2+\frac{C}{Ca}\right)\left(1+\frac{C}{Ca}\right) = \frac{R_P}{K^2Q_1Q_2-1} \quad \ldots(8)$$

Letting $A = \frac{L_1}{rC}\left(2+\frac{C}{Ca}\right)\left(1+\frac{C}{Ca}\right)$, eq.(8) takes the more compact form:

$$A = \frac{R_P}{K^2Q_1Q_2-1}$$

Solving for K we obtain:

3dB bandwidth calculations

We shall now proceed to calculate the 3dB bandwidth of circuit of Fig.1.b.
Mesh current is given by:

$$I = \frac{E_a}{R_e + j\left(\frac{\omega L_1}{1-\omega^2 L_1 C} - \frac{1}{\omega C_T}\right)}$$

where:

$$C_T = \frac{CaC}{Ca+C}$$

The amplitude-frequency relationship for I is determined by:

$$I(\omega) = \frac{E_a}{\sqrt{R_e{}^2 + \left(\frac{\omega L_1}{1-\omega^2 L_1 C} - \frac{1}{\omega C_T}\right)^2}}$$

At the -3dB points:

$$I(\omega) = \frac{E_a}{\sqrt{2}R_e}$$

The corresponding frequencies must satisfy the equation:

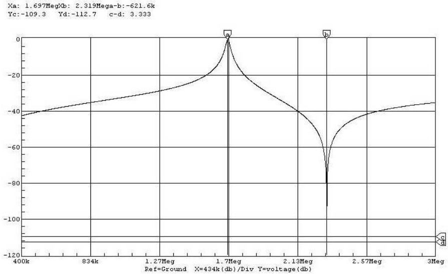

Fig.3.e Frequency response for circuit of Fig.1.b when resonance is adjusted to 1.7MHz

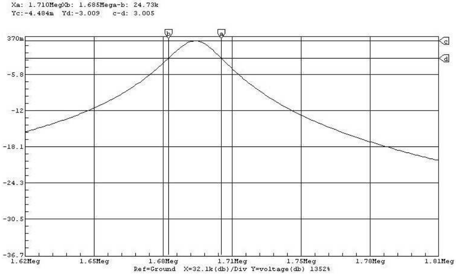

Fig.3.f 3dB bandwidth for a resonance frequency of 1.7MHz is 24.73kHz

$$K = \sqrt{\frac{R_P + A}{Q_1 Q_2 A}} \qquad \ldots(9)$$

which gives the value of the coupling coefficient for maximum RF power transfer to the secondary.

Power calculations

The RF power delivered to the secondary load of Fig.1 will be at a maximum at resonance when eq.(6) is satisfied, this is, when $r + R_s = R_{s1}$. The maximum available power is then:

$$P_{MAX} = \frac{\left(\frac{E_a}{2}\right)^2}{2R_{s1}}$$

$$= \frac{E_a{}^2}{8R_{s1}} \qquad \ldots(10)$$

where E_a is the peak value of the voltage induced in the antenna.

The power delivered to the secondary load R_2 is the same as that dissipated by the coupled resistance N^2R_2. To compute this power we need the voltage across L_1 at resonance. This is the same as the voltage across L_{eq} in Fig.6.a. Then:

$$E_{Leq} = \frac{E_a}{2R_{s1}} j\omega_r L_{eq} \qquad \ldots(11)$$

From Fig.5 we obtain for the voltage across N^2R_2:

$$E_2' = \frac{E_{Leq}}{j\omega_r L_1 + N^2 R_2} N^2 R_2$$

In crystal sets, $N^2 R_2 << \omega_r L_1$, due to the loose coupling between L_1 and L_2. Then:

$$E_2' = \frac{E_{Leq}}{j\omega_r L_1} N^2 R_2$$

$$= \frac{E_{Leq}}{j\omega_r L_1}\left[\frac{(\omega_r L_1)^2}{R_{P1}}\right]$$

$$= \frac{E_{Leq}}{jR_{P1}} \omega_r L_1$$

Bearing in mind eq.(3):

$$E_2' = \frac{E_{Leq}}{j} K^2 Q_2$$

Substituting the value of E_{Leq} given by eq.(11) into the above expression we obtain:

$$E_2' = \frac{E_a}{2R_{s1}} \omega_r L_{eq} K^2 Q_2$$

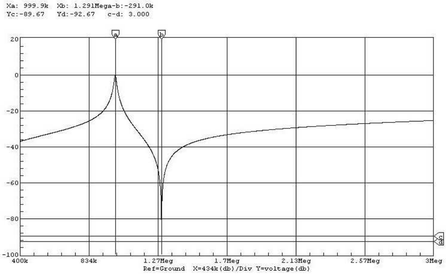

Fig.3.c Frequency response for circuit of Fig.1.b when resonance is adjusted to 1MHz

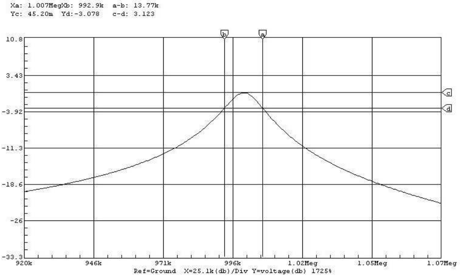

Fig.3.d 3dB bandwidth for a resonance frequency of 1MHz is 13.77kHz

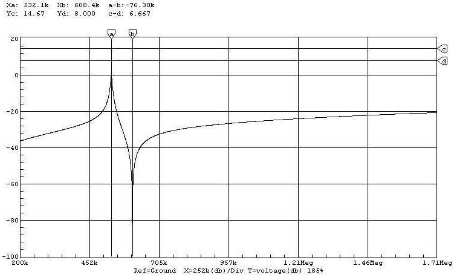

Fig.3.a Frequency response for circuit of Fig. 1.b when resonance is adjusted to 530kHz

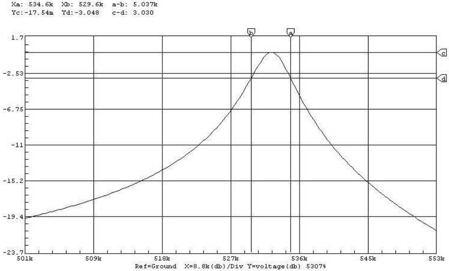

Fig.3.b 3dB bandwidth for a resonance frequency of 530kHz is 5.037kHz

We can recall that:

$$R_{s1} = \left(\omega_r L_{eq}\right)^2 \frac{K^2 Q_1 Q_2}{R_P} \qquad \ldots(12)$$

Then:

$$\frac{\omega_r L_{eq} K^2 Q_2}{R_{s1}} = \frac{R_P}{\omega_r L_{eq} Q_1}$$

$$= \frac{L_1}{L_{eq}}$$

$$= \frac{Ca}{2Ca + C}$$

Then, we obtain:

$$E_2' = \frac{E_a}{2}\left(\frac{Ca}{2Ca + C}\right)$$

The power dissipated by N^2R_2 is:

$$P_2 = \frac{\left(E_2'\right)^2}{2N^2 R_2}$$

$$= \frac{\left(\frac{E_a}{2}\right)^2 \left(\frac{Ca}{2Ca + C}\right)^2}{2N^2 R_2} \quad \ldots(13)$$

Eq.(12) can be written as follows:

$$R_{s1} = (\omega_r L_{eq})^2 \left(\frac{N^2 L_2}{L_1}\right)\left(\frac{Q_1 Q_2}{R_P}\right)$$

$$= (\omega_r L_{eq})^2 \left(\frac{N^2 L_2}{L_1}\right)\left(\frac{1}{\omega_r L_1}\right)\left(\frac{R_2}{\omega_r L_2}\right)$$

$$= \left(\frac{L_{eq}}{L_1}\right)^2 N^2 R_2$$

$$= \left(\frac{2Ca + C}{Ca}\right)^2 N^2 R_2$$

or:

$$N^2 R_2 = \left(\frac{Ca}{2Ca + C}\right)^2 R_{s1}$$

Substituting this equivalence into eq.(13):

$$P_2 = \frac{E_a^{\ 2}}{8R_{s1}} = P_{MAX}$$

according to eq.(10).

Figures 3.a through 3.f show simulation results for circuit of Fig.1.b at resonance frequencies also of 530kHz, 1MHz and 1.7MHz. Again, r = 30 ohms and Ca = 200pF.
Re = $2Rs_1$, and it can be easily shown that:

$$R_{s1} = r\left(1 + \frac{A}{R_P}\right)$$

The Y axis on the graphics represents voltage across Re in decibels, with Ea being a 1 volt-amplitude unmodulated carrier.

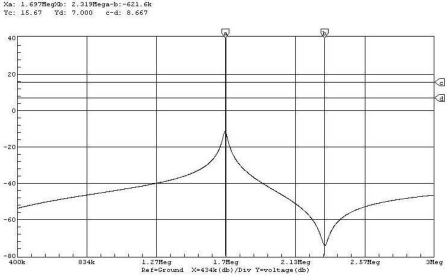

Fig.2.e Frequency response of circuit of Fig.1.a when resonance is adjusted to 1.7MHz

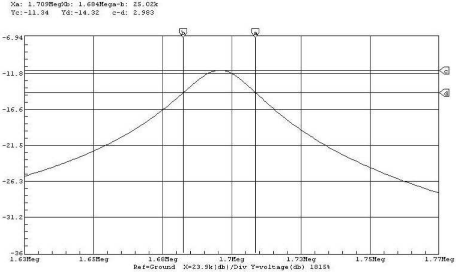

Fig.2.f 3dB bandwidth for a resonance frequency of 1.7MHz is 25.02kHz

P_{MAX} is then dissipated by N^2R_2 and by consequence, this power is delivered to the secondary load.

Some experimental results

Two coils, L_1 and L_2, were wound on 4.5" diameter styrene forms using 660/46 Litz wire. L_1 measured 152 uH and L_2, 222 uH. A two-gang 475 pF variable capacitor with bakelite insulation was used to tune L_1. L_2 was tuned with a 480 pF variable capacitor with ceramic insulators.
Unloaded Qs for each of the tuned circuits were measured at three frequencies. Accordingly, the corresponding RF losses were calculated. Data is tabulated below.

f	Q_1	Q_{2UL}	Q_2	R_{Pcalc}	$R_{TNKcalc}$	C	C_2
530kHz	700	810	270	354.32 kohms	598.816 kohms	450 pF	406 pF
1 MHz	585	630	210	558.70 kohms	878.766 kohms	100 pF	114 pF
1.7 MHz	300	318	106	487.072 kohms	754.065 kohms	31 pF	39.5 pF

C : two-gang 475 pF variable capacitor with bakelite insulation
C_2 : 480 pF variable capacitor with ceramic insulation
Q_{2UL} : unloaded Q of L_2-C_2 combination
Q_1, Q_2 : defined in the text
R_P, R_{TNK}: defined in the text

Using the tabulated data, values for the optimum coupling coefficient K will be calculated for a working crystal set.

f = 530 kHz
L_1 = 152 uH
Ca = 200 pF (assumed)
r = 30 ohms (assumed)
A = 1.555 x 10^5 ohms
Q_1Q_2A = 2.938 x 10^{10} ohms
K = 4.165 x 10^{-3}
Check:

$$\frac{R_P}{1+K^2Q_1Q_2} = 82.8kohms >> \omega_r L_{eq} = 2.214kohms$$

f = 1 MHz
L_1 = 152 uH
Ca = 200 pF (assumed)
r = 30 ohms (assumed)
A = 1.9 x 10^5 ohms
Q_1Q_2A = 2.334 x 10^{10} ohms
K = 5.663 x 10^{-3}
Check:

$$\frac{R_P}{1+K^2Q_1Q_2} = 113.1kohms >> \omega_r L_{eq} = 2.387kohms$$

f = 1.7 MHz
L_1 = 152 uH
Ca = 200 pF (assumed)
r = 30 ohms (assumed)
A = 4.068 x 10^5 ohms
Q_1Q_2A = 1.29 x 10^{10} ohms
K = 8.324 x 10^{-3}

Check:

$$\frac{R_P}{1+K^2Q_1Q_2} = 152kohms >> \omega_r L_{eq} = 3.498kohms$$

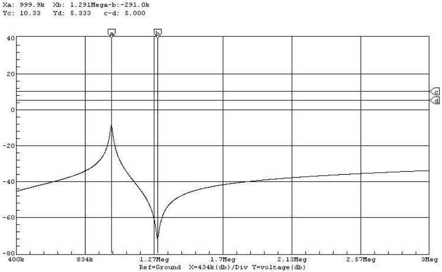

Fig.2.c Frequency response of circuit of Fig.1.a
when resonance is adjusted to 1MHz

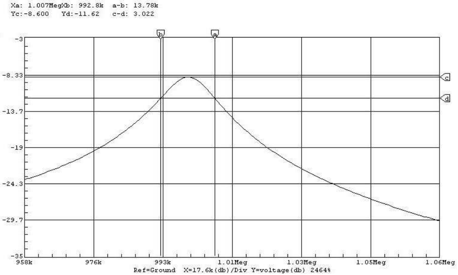

Fig.2.d 3dB Bandwidth for a resonance
frequency of 1MHz is 13.78kHz

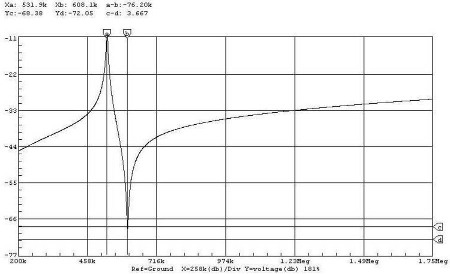

Fig.2.a Frequency response of circuit of Fig.1.a
when resonance is adjusted to 530kHz

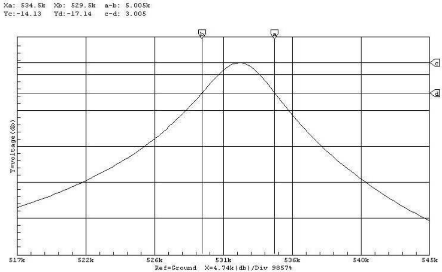

Fig.2.b 3dB bandwidth for a nominal
resonance frequency of 530kHz is 5.005kHz.
Actually, the resonance frequency is 532kHz.

Comments:

The values obtained for the coupling coefficient K hold for $R_2 = \frac{R_{TNK}}{3}$, as discussed previously. The transformed antenna-ground system resonance resistance, as seen from the secondary, will be equal to R_2, or $\frac{R_{TNK}}{3}$, as we are dealing with maximum power transfer to R_2. Under these conditions, the loaded Q of the secondary circuit will be 1/6 of the unloaded value. For other load conditions, the respective data should be entered into eq.(9).

Analysis of the Tuggle Front End – Part III

As a first approximation, the 3dB bandwidth of the antenna-ground-lossy tuner system under matched load conditions can be computed assuming that the L_1-C tank behaves as an equivalent constant inductance Leq in the 3dB passband, this inductance being in series with the rest of the circuit. However, this approach leads to large errors in the results, as suggested by a SPICE circuit simulation.

A precise model for accurate bandwidth computation is shown in Fig.1.a below. R_T is the net RF resistance in parallel with the L_1-C tank at $\omega = \omega_r$, as found in part II of our study. Ground losses Rg and antenna radiation resistance r_a are accounted for by r. However, calculations on this circuit are rather tedious. Simulation shows that the circuit depicted in Fig.1.b can be alternatively employed for bandwidth computation with equivalent results to those given by the circuit of Fig.1.a. Here, Re = $2Rs_1$ (please refer to part II). Circuit calculations in this case are much more simple.

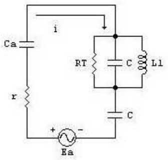

Fig.1.a Equivalent circuit for accurate 3dB bandwidth calculation

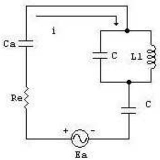

Fig.1.b Alternative circuit for bandwidth calculation as suggested by simulation

Simulation results

Figures 2.a through 2.f illustrate simulation results for circuit of Fig.1.a at resonance frequencies of 530kHz, 1MHz and 1.7MHz. Assumed values for r and Ca are 30 ohms and 200pF, respectively. The values for R_T are those obtained when the secondary load R_2 is impedance matched to the primary side (please refer to part II). Notch frequencies occurring above resonance can be observed on the graphics. The Y axis represents voltage across r in decibels with Ea being a 1volt-amplitude unmodulated carrier.